Edexcel Maths M4

Mark Scheme Pack

2002-2015

EDEXCEL FOUNDATION

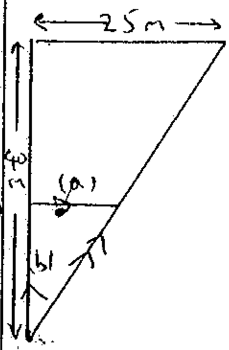
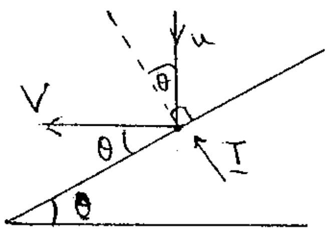
Stewart House 32 Russell Square London WC1B 5DN

January 2002

Advanced Subsidiary /Advanced Level

General Certificate of Education

Subject **MECHANICS 6680** Paper No. **M4**

Question number	Scheme	Marks	
1. (a)	 Complete method for speed of current	M1	
	e.g $= \frac{25\text{m}}{30\text{s}}$ or find $V(1.57)$, $\theta(32°)$ and use $V\sin\theta$ or equiv.		
	$= \frac{5}{6}$ ms^{-1} or 0.83(3) ms^{-1}	A1	(2)
(b)	Complete method for speed of swimmer	M1	
	e.g. $= \frac{40\text{m}}{30\text{s}}$ or $\sqrt{\{V^2 - (a)^2\}}$ or $V_c \sin\theta_c$		
	$= \frac{4}{3}$ ms^{-1} or 1.3(3) ms^{-1}	A1	(2)
2.	Equation of motion: $-mg - mkv = ma$; $\frac{dv}{dt} = -(g + kv)$	M1;A1	
	Separating variables: $\int dt = -\int \frac{dv}{g+kv}$	M1	
	Integrating $t = (-)\frac{1}{k}[\ln(g+kv)]$ (+c)	A1	
	Using limits to give $T = \frac{1}{k}[\ln(g+kv)]_0^u$ or using limits [t=0,v=u] to find c:	M1A1	
	Completing to give $T = \frac{1}{k}\ln(\frac{g+ku}{g})$	M1A1	(8)
	[Mark finding greatest height as Mr]		
3. (a)	 Parallel to plane: $u \sin\theta = V\cos\theta$	M1A1	
	Perpendicular to plane: $e\,u\cos\theta = V\sin\theta$	M1A1	
	Eliminating u and V: $e\cot\theta = \tan\theta$	M1	
	Given result: $e = \tan^2\theta$ *	A1	(6)
(b)	Impulse = change in momentum $= m(V\sin\theta + u\cos\theta)$	M1A1	
	Expression in m, u and θ: $= m(e\,u\cos\theta + u\cos\theta) = mu\cos\theta(1 + \tan^2\theta)$	M1	
	or $= mu\left(\frac{\sin^2\theta}{\cos\theta} + \cos\theta\right)$		
	Completion $= mu\sec\theta$ *	A1	(4)

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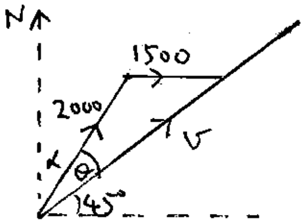
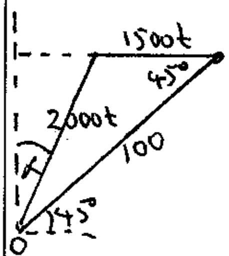
4. (a)	*Using velocity diagram* $\frac{\sin\theta}{1500} = \frac{\sin 45°}{2000}$	M1A1
		
	$\theta = 32°$ (32.03)	M1A1
	Bearing $= 90° - (45° + \theta) = 013°$	M1A1 (6)
(b)	Method for v : e.g. (i) $v^2 = 1500^2 + 2000^2 - 2.1500.2000.\cos(90 + 13_c)°$	M1A1√
	or (ii) $v \cos 45° = 2000 \cos 13_c°$	
	or (iii) $\frac{\sin 45°}{2000} = \frac{\sin 103°}{v}$	
	$v = 2756$ km h^{-1}	A1
	Time $= \frac{100}{v}$ h $= 131$ *s*	M1A1 (5)
	[Time $= \frac{100 \cos 45°}{2000 \cos 13_c°}$ gains M1M1A1 immediately, correct answer gains A2]	
	Using displacement method (several variations)	
	(i) In the case below α is bearing; but other relevant angle may be used	
		
	One equation in t and α: e.g. $2000\,t \sin\alpha = 50\sqrt{2} - 1500\,t$	M1A1
	Second equation in t and α: e.g. $2000\,t \cos\alpha = 50\sqrt{2}$	M1A1
	Equation in one variable: e.g. $4\cos\alpha - 4\sin\alpha = 3$	M1A1
	Reducing to simple equation e.g. $4\sqrt{2}\cos(\alpha + 45°) = 3$	M1A1√
	Bearing = (0)13°	A1
	Sustituting for α to find t; $t = 131$ *s*	M1A1
	(ii) Using cosine rule: $(2000t)^2 = (1500t)^2 + 100^2 - 2.100.1500t \cos 45°$	M2A1A1
	Quadratic form: $175t^2 + 15\sqrt{2}\,t - 1 = 0$	M1A1 √
	Solving : $t = 131s$	M1A1
	Equation in t and α	M1A1
	Bearing = (0)13°	A1

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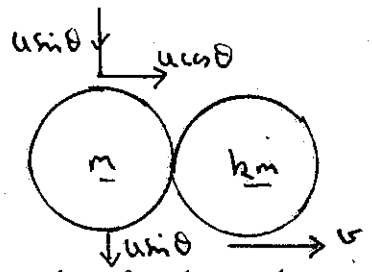
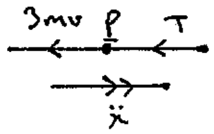
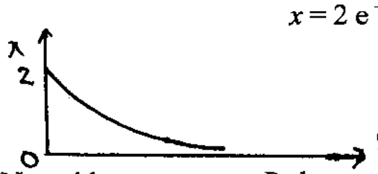
Question number	Scheme	Marks	
5. (a)	 CLM: $mu\cos\theta = kmv$	M1A1	
	NIL: $eu\cos\theta = v$	M1A1	
	Eliminating θ,	M1	
	$e = \frac{1}{k}$ *	A1	(6)
(b)	$\frac{1}{2}m\,v_a^2 + \frac{1}{2}(2m)(\frac{1}{2}u\cos\theta)^2 = \frac{3}{4}.\frac{1}{2}mu^2$ (or equivalent)	M1A1	
	$\frac{1}{2}m(u\sin\theta)^2 + \frac{1}{2}(2m)(\frac{1}{2}u\cos\theta)^2 = \frac{3}{4}.\frac{1}{2}mu^2$ [M1 for $v_a = u\sin\theta$]	M1A1√	
	$[4\sin^2\theta + 2\cos^2\theta = 3]$		
	$4\sin^2\theta + 2(1-\sin^2\theta) = 3$	M1	
	$\sin^2\theta = \frac{1}{2}$		
	$\theta = 45°$	A1	(6)
	[$\frac{1}{2}m(u\cos\theta)^2 - \frac{1}{2}(2m)(\frac{1}{2}u\cos\theta)^2 = \frac{1}{4}\frac{1}{2}mu^2$ accepted for first 4 marks unless it is clear that candidate is working along line of centres only; e.g. $\frac{1}{2}m(u\cos\theta)^2 - \frac{1}{2}(2m)(\frac{1}{2}u\cos\theta)^2 = \frac{1}{4}\frac{1}{2}m(u\cos\theta)^2$, then max M1]		
6. (a)	 $T = \frac{2mL}{L}x$	B1	
	Equation of motion: $-3m\dot{x} - T = m\ddot{x}$	M1A1	
	$\Rightarrow \ddot{x} + 3\dot{x} + 2x = 0$ *	A1 (cso)	(4)
(b)	A.E. $m^2 + 3m + 2 = 0 \Rightarrow m = -1$ or -2	M1A1	
	G.S. $x = Ae^{-t} + Be^{-2t}$	A1√	
	$t = 0, x = 2: \Rightarrow A + B = 2$	B1	
	Differentiating $\dot{x} = -Ae^{-t} - 2Be^{-2t}$	M1	
	$t = 0, \dot{x} = -4: \Rightarrow A + 2B = 4$ (any equivalent form)	A1	
	Correctly solving simultaneous equations ($A = 0$, $B = 2$)	M1	
	$x = 2e^{-2t}$	A1	(8)
(c)	 Shape	B1√	
	(0,2), $x = 0$ asymptote Totally correct	B1	(2)
(d)	No, with reason, e.g. P always moving	B1	(1)

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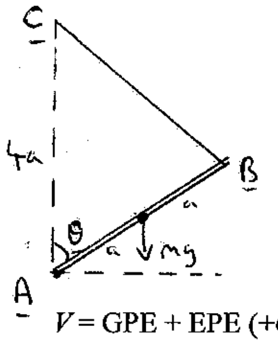
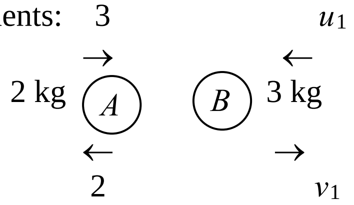
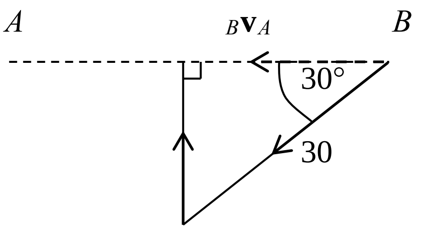
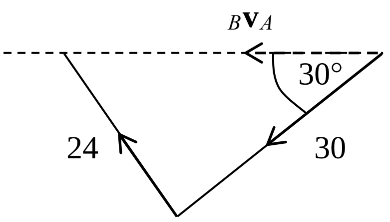
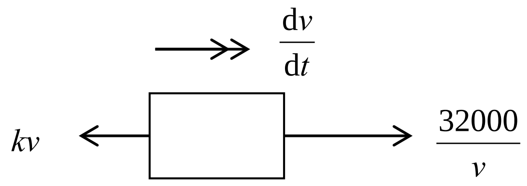


Question number	Scheme	Marks	
7. (a)	GPE: (from a fixed point) e.g. $mga\cos\theta$ (+C)	M1	
	EPE: $\frac{1}{2}mg\frac{(\text{ext})^2}{4a}$	B1	
	$BC^2 = (4a)^2 + (2a)^2 - 2.4a.2a.\cos\theta = 20a^2 - 16a^2\cos\theta$	M1 A1	
	$\Rightarrow$ EPE $= \frac{1}{2}mga[\,5 - 4\cos\theta - 2\{\sqrt{(5-4\cos\theta)}\} + 1]$	M1 A1	
	V = GPE + EPE (+C) applied	M1	
	$= mga\{-\cos\theta - \sqrt{(5-4\cos\theta)} + 3\} + C$ ($\sqrt{}$ dep. on all Ms)	A1$\sqrt{}$	
	$= mga\{-\cos\theta - \sqrt{(5-4\cos\theta)}\}$ + constant * (no errors seen)	A1	(9)
(b)	$\frac{dV}{d\theta} = mga\left\{\sin\theta - \frac{4\sin\theta}{2\sqrt{5-4\cos\theta}}\right\}$	M1A1	
	$\frac{dV}{d\theta} = 0$; $[\sin\theta\{\sqrt{(5-4\cos\theta)} - 2\} = 0]$	M1	
	$\Rightarrow \sin\theta = 0$ or $\sqrt{(5-4\cos\theta)} = 2$		
	$\Rightarrow \theta = 0$ or π (0° or 180°)	A1	
	$\Rightarrow$ or $\theta = \cos^{-1}(\frac{1}{4}) = 1.32$ (75.5°)	M1A1	(6)

Question Number	Scheme	Marks
1.	**i** components: 	
	Momentum ⟷: $2 \times 3 - 3u_1 = -2 \times 2 + 3v_1$	M1
	NLI: $10 = 3u_1 + 3v_1$	A1
	$\frac{1}{2}(3 + u_1) = 2 + v_1$	M1
	$1 = u_1 - 2v_1$	A1
	Solve for u_1, $23 = 9u_1$ $\Rightarrow u_1 = \frac{23}{9}$	M1 A1
	j component $= -u_1 \tan\alpha = -\frac{46}{9}$	M1 A1 ft
	Hence $\mathbf{u}_B = -\frac{23}{9}\mathbf{i} - \frac{46}{9}\mathbf{j}$	A1
		(9 marks)
2.	 ${}_B\mathbf{v}_A$ as ←	M1
	Correct △ for $\mathbf{v}_B$ minimum	M1
	$v = 30 \sin 30° = 15 \text{ km h}^{-1}$	A1 (3)
	 Correct △	M1
	$\frac{30}{\sin\alpha} = \frac{24}{\sin 30°}$	M1
	$\Rightarrow \sin\alpha = \frac{5}{8}$	A1
	$\lvert {}_B\mathbf{v}_A \rvert = 30\cos 30° + 24\cos\alpha$ (≈ 44.716)	M1 A1
	$T = \frac{20}{44.716} \approx 0.4473$	M1
	$\Rightarrow$ 0927 hrs (awrt)	A1 (7)
		(10 marks)

(ft = follow through mark; awrt = anything which rounds to)

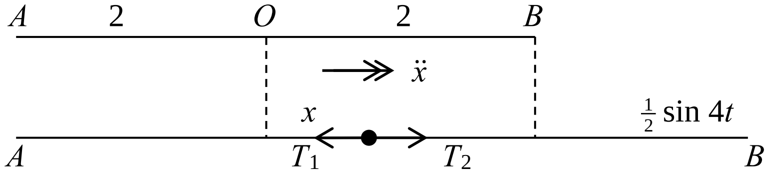
Question Number	Scheme	Marks
3. (*a*)		
	$800\,\frac{dv}{dt} = \frac{32000}{v} - kv$	M1 A1
	$\Rightarrow\ 800v\frac{dv}{dt} = 32000 - kv^2$ (*)	A1 (3)
(*b*)	$v = 40,\ \frac{dv}{dt} = 0 \Rightarrow 32000 = k \times 40^2$	M1
	$\Rightarrow k = 20$	A1 (2)
(*c*)	$\int dt = 800\int \frac{v\,dv}{32000 - 20v^2} = \int \frac{40v\,dv}{1600 - v^2}$	M1
	$t = -20 \ln (1600 - v^2)$ $(+ C)$	M1 A1 ft
	$t = 0,\ v = 0 \Rightarrow C = 20 \ln 1600$ (or use of limits)	
	$t = 20 \ln 1600 - 20 \ln (1600 - v^2)$	M1 A1 ft
	$\Rightarrow\ t = 20 \ln \left(\frac{1600}{1600 - v^2}\right)$	
	$\frac{1600}{1600 - v^2} = e^{\frac{t}{20}}$	M1
	$1600\,e^{-\frac{t}{20}} = 1600 - v^2$	
	$v = 40\sqrt{\left(1 - e^{-\frac{t}{20}}\right)}$	A1 (7)
		(12 marks)

((*) indicates answer is given on the examination paper)

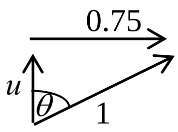
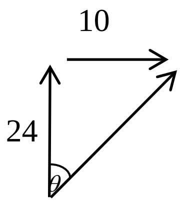
Question Number	Scheme	Marks
4. (*a*)	$AC = 4a\cos\theta \Rightarrow$ extension in spring $= 4a\cos\theta - 2a$	B1
	$V = -2mga\cos\theta - 2mg \times 3a\cos\theta + \frac{4mg}{4a}(4a\cos\theta - 2a)\ (+C)$	M1 A1 A1
	$= -8mga\cos\theta + 4mg(2\cos\theta - 1)^2\ (+C)$	
	$= 4mga[(2\cos\theta - 1)^2 - 2\cos\theta]\ (+C)$	A1 (5)
(*b*)	$\frac{dV}{d\theta} = 4mga[2(2\cos\theta - 1)(-2\sin\theta) + 2\sin\theta)]$	M1 A1
	$= 8mga\sin\theta(3 - 4\cos\theta)$	
	$= 0 \Rightarrow \cos\theta = \frac{3}{4}\ (\theta \neq 0, \pi)$	M1 A1
	$\Rightarrow \theta = 0.723$	A1 (4)
(*c*)	$\frac{d^2V}{d\theta^2} = 8mga\cos\theta(3 - 4\cos\theta) + 32mga\sin^2\theta$	M1 A1
	as $\theta = \frac{3}{4} \Rightarrow \frac{d^2V}{d\theta^2} = 0 + 32mga \times \frac{7}{16}$	M1
	$= 14mga$	
	$> 0 \Rightarrow$ stable	A1 (4)
		(13 marks)

Question Number	Scheme	Marks
5. (*a*)	$\mathbf{r}_P = \begin{pmatrix}1\\-2\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix} \quad \mathbf{r}_Q = \begin{pmatrix}-1\\2\\-1\end{pmatrix} + t\begin{pmatrix}2\\0\\1\end{pmatrix}$	B1 (either)
	$\begin{pmatrix}1\\-2\\3\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix} + t\begin{pmatrix}2\\0\\1\end{pmatrix}$	M1
	$\Rightarrow t = 2$ (one component)	A1
	showing true for all components $\Rightarrow$ collide	M1
	$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1 (5)
(*b*)	$\mathbf{v}_R - \mathbf{v}_P = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix}, \quad \mathbf{v}_R - \mathbf{v}_Q = \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	B1
	$\mathbf{v}_Q - \mathbf{v}_P = \begin{pmatrix}2\\0\\1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix} - \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	M1
	$\begin{pmatrix}1\\-2\\2\end{pmatrix} = \lambda\begin{pmatrix}-5\\4\\-1\end{pmatrix} - \mu\begin{pmatrix}-2\\2\\-1\end{pmatrix}$	
	$\left.\begin{array}{r}-5\lambda + 2\mu = 1\\ 4\lambda - 2\mu = -2\\ -\lambda + \mu = 2\end{array}\right\}$	A1
	Solve for either $\lambda = 1$ or $\mu = 3$	M1 A1
	Hence $\mathbf{v}_R = -4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$	A1
	$\begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix} + 2\begin{pmatrix}-4\\6\\-2\end{pmatrix}$	M1 A1ft
	$\Rightarrow a = 11, b = -10, c = 5$	
	$t = 0$, R is at $11\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}$	A1 (9)
		(14 marks)

(ft = follow through mark)

Question Number	Scheme	Marks
6. (*a*)	$t = 0$ 	
	$T_1 = 4(1 + x)$; $T_2 = 4(1 + \frac{1}{2}\sin 4t - x)$	B1; B1
	$T_2 - T_1 = 2\ddot{x}$	M1
	$2\ddot{x} = 4(1 + \frac{1}{2}\sin 4t - x) - 4(1 + x)$	A1
	$\Rightarrow \ddot{x} + 4x = \sin 4t$ (✱)	A1 (5)
(*b*)	CF: $x = A\sin 2t + B\cos 2t$	B1
	PI: $x = P\sin 4t$	M1
	$-16P\sin 4t + 4P\sin 4t = \sin 4t$	M1
	$\Rightarrow P = -\frac{1}{12}$	A1
	$x = A\sin 2t + B\cos 2t - \frac{1}{12}\sin 4t$	M1
	$t = 0, x = 0 \Rightarrow B = 0$	A1
	$\dot{x} = 2A\cos 2t - \frac{1}{3}\cos 4t$	M1
	$t = 0, \dot{x} = 0 \Rightarrow A = \frac{1}{6}$	A1 ft (8)
	$\dot{x} = 0 \Rightarrow \frac{1}{3}\cos 2t - \frac{1}{3}\cos 4t = 0$	M1
	$\Rightarrow \cos 4t = \cos 2t$	
	$\Rightarrow 4t = 2t + 2\pi$ *or* $2\pi - 2t$	M1
	$\Rightarrow t = \pi$ or $\frac{\pi}{3}$	A1
	$\Rightarrow$ First at rest after $t = 0$ when $t = \frac{\pi}{3}$	A1 cso (4)
		(17 marks)

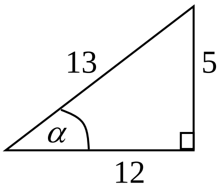
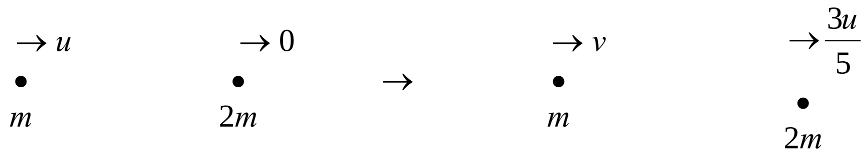
(ft = follow through mark; (✱) denotes answer is given on paper; cso = correct solution only)

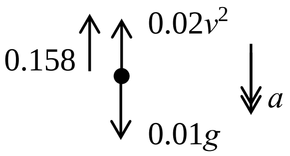
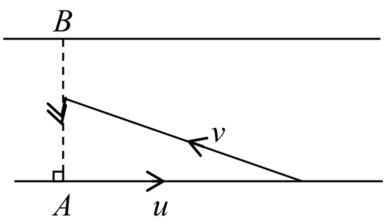
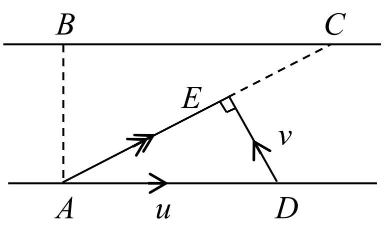
Question Number	Scheme	Marks
1.	Let boy's velocity be $\uparrow u$, $\rightarrow 0.75$	M1
	Speed = 1 $\Rightarrow$ $1^2 = u^2 + \frac{9}{16}$, $\therefore u^2 = \frac{7}{16}$ or $u = \frac{\sqrt{7}}{4}$ or 0.661…	M1 A1
	Time = $\frac{100}{\sqrt{7}/4}$ = 151.18… = 151s	A1
		
	$\sin\theta = \frac{0.75}{1}$ $\Rightarrow \theta = 48.6$	M1
	$\therefore$ Bearing is 049° or 048.6°	A1
		(6)
		(6 marks)
2.	Let wind be $\uparrow W_y$, $\rightarrow W_x$	M1
	Relative to A: $\uparrow W_y$, $\rightarrow W_x - 10$ From South, $\Rightarrow$ $W_x = 10$	M1, A1
	Relative to B: $\uparrow W_y - 14$, $\rightarrow W_x$ From SW, $\Rightarrow W_y - 14 = W_x$ $\therefore W_y = 24$	M1, A1
	$\therefore$ Magnitude of $W = \sqrt{10^2 + 24^2}$ = 26 km h^{-1}	A1
		
	$\tan\alpha = \frac{10}{24} \Rightarrow \alpha = 22.6$	
	$\therefore$ Bearing 023° or 022.6°	A1
		(7 marks)

Question Number	Scheme	Marks
3.	(↓) $mg - mkv^2 = ma$	M1 A1
	$g - kv^2 = v\frac{dv}{dx}$ $v\frac{dv}{dx}$	M1
	$x = \int \frac{v}{g - kv^2}\,dv$	M1
	$x = -\frac{1}{2k}\ln\left\|g - kv^2\right\| + c$	M1 A1
	$x = 0,\ v = 0 \Rightarrow 0 = -\frac{1}{2k} + c$	M1
	$x = \frac{1}{2k}\ln\left\|\frac{g}{g - kv^2}\right\|$	
	$e^{2kx} = \frac{g}{g - kv^2}$	A1
	$kv^2 = g(1 - e^{-2kx})$	M1
	$v = \sqrt{\frac{g}{k}\left(1 - e^{-2kD}\right)}$ must use D	A1
		(11 marks)

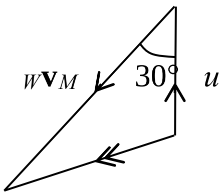
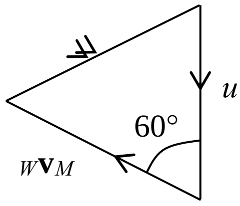
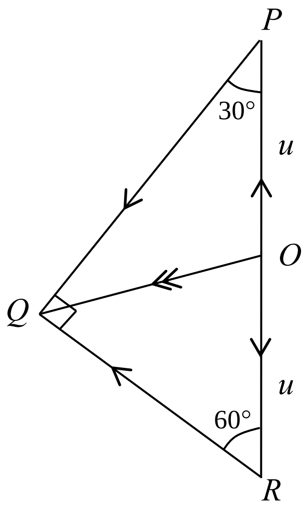
Question Number	Scheme	Marks
4. (*a*)	P.E.of rod = $mg \times 2a \sin 2\theta$	B1
	$AC = a \cot \theta$	B1
	EPE in String = $\frac{1}{2} \times \frac{3}{4} \times \frac{mg}{a}(a \cot \theta - a)^2$	M1 A1
	Total P.E $V = mg.2a \sin 2\theta + \frac{3}{8}\frac{mg}{a}(a \cot \theta - a)^2$	M1
	$= \frac{mga}{8}[16 \sin 2\theta + 3 \cot^2 \theta - 6 \cot \theta + 3]$	M1
	i.e. $V = \frac{mga}{8}[16 \sin 2\theta + 3\cot^2 \theta - 6 \cot \theta] + \text{const}$ (*)	A1 cso (7)
(*b*)	$\frac{\mathrm{d}v}{\mathrm{d}\theta} = \frac{mga}{8}[32 \cos 2\theta - 6 \cot \theta \operatorname{cosec}^2 \theta + 6 \operatorname{cosec}^2 \theta]$	M1 A2, 1, 0
	$\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\vert_{\theta=0.535} = \frac{mga}{8}(-0.5^{0.1\ldots}_{\ldots\ldots})$	M1
	$\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\vert_{\theta=0.545} = \frac{mga}{8}(0.2^{99\ldots}_{\ldots\ldots})$	A1
	Change of sign $\therefore \frac{\mathrm{d}v}{\mathrm{d}\theta} = 0$ in range, so $\exists$ find a position of equilibrium	A1 (6)
	(c) $\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\vert_{0.535} < 0$, $\left.\frac{\mathrm{d}v}{\mathrm{d}\theta}\right\vert_{\theta=0.545} > 0$	M1
	So turning point is *minimum*, $\therefore$ equilibrium is *stable*	A1, A1 (3)
		(16 marks)

Question Number	Scheme	Marks
5. (*a*)	Auxiliary Equation.: $m^2 + 2m + 2 = 0, \Rightarrow m = -1 \pm \text{i}$	M1, A1
	$\therefore$ Complementary. Function is: $x = e^{-t}(\text{A} \cos t + \text{B} \sin t)$	M1 ft
	Let $x = p \cos 2t + q \sin 2t$, $\dot{x} = -2p \sin 2t + 2q \cos 2t$, $\ddot{x} = -4x$	M1
	Sub. in D.E.	
	$-2p \cos 2t - 2q \sin 2t - 4p \sin 2t + 4q \cos 2t = 12 \cos 2t - 6 \sin 2t$	M1
	$-2p + 4q = 12, -4p - 2q = -6$	A1
	$-10p = 0 \Rightarrow p = 0, q = 3$	M1
	$\therefore$ $x = 3 \sin 2t + e^{-t}(\text{A} \cos t + B \sin t)$	A1
	$t = 0, x = 0 \Rightarrow 0 = A$	B1
	$\dot{x} = 6 \cos 2t - e^{-t} B \sin t + e^{-t} B \cos t$	M1
	$t = 0, \dot{x} = 0 \Rightarrow 0 = 6 + B \quad \therefore B = -6$	
	$\therefore$ $x = 3 \sin 2t - 6e^{-t} \sin t$	A1 (11)
(*b*)	$\dot{x} = 6[\cos 2t + e^{-t} \sin t - e^{-t} \cos t]$	
	Sub $t = \frac{\pi}{4}$ $\dot{x} = 6[\cos 2t + e^{-t} - 6e^{-t} \cos t]$	
	$\dot{x} = 6\left[0 + e^{-\frac{x}{4}} \times \frac{1}{\sqrt{2}} - e^{-\frac{x}{4}} \times \frac{1}{\sqrt{2}}\right] = 0$	M1
	$\therefore$ *P* comes to instantaneous rest when $t = \frac{\pi}{4}$	A1 (2)
(*c*)	sub $t = \frac{\pi}{4}$ in x $= 3 \sin \frac{\pi}{2} - 6e^{-\frac{\pi}{4}} \frac{1}{\sqrt{2}}, = 1.07$	M1, A1 (2)
(*d*)	$t \to \infty$ $x \approx 3 \sin 2t$, approximate period is π	M1, A1 (2)
		(17 marks)

Question Number	Scheme	Marks
6. (*a*)	 P before: $\rightarrow \frac{13u}{12}\cos\alpha = u$, $\uparrow \frac{13u}{12}\sin\alpha = \frac{5u}{12}$	B1, B1
		
	PCLM ($\rightarrow$) $mu = mv + 2m\frac{3u}{5}, \Rightarrow v = \frac{-u}{5}$, i.e. $\frac{u}{5}$ // CB	M1 A1 (4)
(*b*)	NLI $\rightarrow$ $eu = v_2 - v_1 \Rightarrow eu = \frac{3u}{5} - -\frac{u}{5}$, i.e. $e = \frac{4}{5}$	M1, A1 (2)
(*c*)	$Q \rightarrow C$ $t_1 = \frac{d_1}{3u/5} = \frac{5d_1}{3u}$	B1
	P travels $\frac{u}{5} \times \frac{5d_1}{3u} = \frac{d_1}{3}$ in direction CB	M1
	$\therefore$ P is $d_1 + \frac{d_1}{3} = \frac{4d_1}{3}$ from w (∗)	A1 c.s.o (3)
(*d*)	After hitting w, Q has speed $\frac{3u}{10}$ in direction CB	B1
	Velocity of Q relative to P in direction CB is $\frac{u}{10}$	M1
	Time for Q to travel $\frac{4}{3}d_1$ is: $\frac{4d_1}{3u} \times 10 = \frac{40d_1}{3u}$	A1
	Total time between collisions is: $\frac{40d_1}{3u} + \frac{5d_1}{3u} = \frac{15d_1}{u}$ (∗)	A1 c.s.o (4)
(*e*)	For collision to occur P must travel $\uparrow d_2$ and $\downarrow d_2$ in time $\frac{15d_1}{u}$	
	$d_2\uparrow$ $t_2 = \frac{d_2}{5u/12} = \frac{12d_2}{5u}$	B1
	$\downarrow d_2$ velocity $\downarrow$ is $\frac{5u}{24}$, $\therefore$ $t_3 = \frac{d_2}{5u/24} = \frac{24d_2}{5u}$	B1, B1
	Total time is $\frac{36d_2}{5u} = \frac{15d_1}{u}$,	M1
	$\therefore$ $12d_2 = 25d_1$, i.e. $d_1{:}d_2 = 12{:}25$	A1 (5)
		(18 marks)

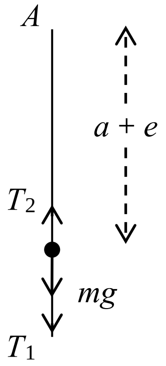
Question Number	Scheme	Marks	
1. (*a*)			
	$0.01a = 0.01g - 0.158 - 0.02v^2$	M1	
	$a = v\dfrac{dv}{dx}$	M1	
	$v\dfrac{dv}{dx} = -2v^2 - 6$ (*)	A1	(3)
(*b*)	$-\displaystyle\int \frac{v\,dv}{2v^2+6} = \int dx$	M1	
	$x = \frac{1}{4}\ln(2v^2+6)\ (+\,C)$	A1	
	$x = 0,\ v = 0 \Rightarrow C = \frac{1}{4}\ln 206$	A1	
	$v = 0 \Rightarrow x = \frac{1}{4}\ln\dfrac{206}{6} \approx 0.884$ m	M1 A1	(5)
			(8 marks)
2. (*a*)			
	vector triangle attempted	M1	
	vector triangle correct	A1	
	Explanation for $v > u$		
	(e.g. 'hypotenuse > other sides')	A1	(3)
(*b*)			
	vector triangle attempted	M1	
	right angle correctly placed	A1	
	$\dfrac{BC}{AB} = \dfrac{AE}{ED}$ Use of similar triangles	M1	
	$= \dfrac{\sqrt{(u^2 - v^2)}}{v}$	M1 A1	(5)
			(8 marks)

((*) indicates final line is given on the paper)

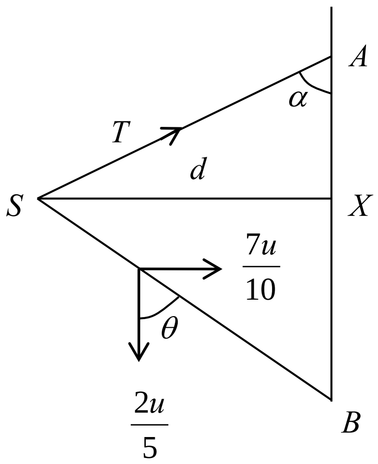
Question Number	Scheme	Marks
3.	$\mathbf{v}_W = {}_W\mathbf{v}_M + \mathbf{v}_M$ (used)	M1
	  (one vector triangle)	M1
		A1 A1
	Combining	M1
		
	$P\hat{Q}R = 90°$	M1
	$QR = 2u \sin 30° = u$	M1
	$\Rightarrow$ triangle OQR is equilateral	
	$\Rightarrow OQ = \mathbf{v}_W = u$	A1
	also $\Rightarrow Q\hat{O}R = 60°$	
	Hence direction is from N 60° E	A1 (9)
		(9 marks)

Question Number	Scheme	Marks	
4. (*a*)	Extension of string = $7a - 2a\cos\theta - a$	B1	
	$= 2a(3 - \cos\theta)$		
	PE = $8mga\cos\theta, + \frac{4mg}{5} \times \frac{4a^2}{2a}(3 - \cos\theta)^2$	B1, M1 A1	
	$= 8mga\cos\theta + \frac{8mga}{5}(9 - 6\cos\theta - \cos^2\theta)$	M1	
	$= \frac{8mga}{5}(\cos^2\theta - \cos\theta) + C$ (*)	A1	(6)
(*b*)	$\frac{dV}{d\theta} = \frac{8mga}{5}(-2\cos\theta\sin\theta + \sin\theta)$	M1 A1	
	$= 0$	M1	
	$\Rightarrow \sin\theta = 0$ *or* $\cos\theta = \frac{1}{2}$		
	$\Rightarrow \theta = 0$ or π, *or* $\theta = \frac{\pi}{3}$	A1, A1	(5)
(*c*)	$\frac{d^2V}{d\theta^2} = \frac{8mga}{5}(\cos\theta + 2\sin^2\theta - 2\cos^2\theta)$	M1 A1	
	$\theta = 0$ $V'' < 0$ $(= -\frac{8mga}{5})$ unstable		
	$\theta = \pi$ $V'' < 0$ $(= -3 \times \frac{8mga}{5})$ unstable	A1	
	$\theta = \frac{\pi}{3}$ $V'' > 0$ $(= 3 \times \frac{8mga}{5})$ stable	A1	(4)
		(15 marks)	

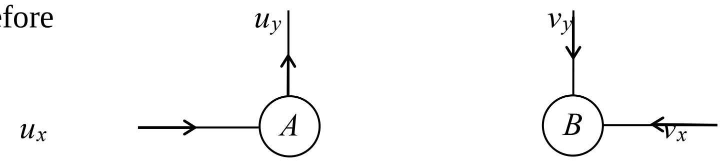
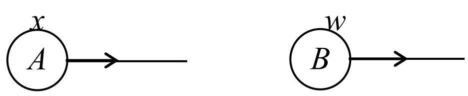
((*) indicates final line is given on the paper)

Question Number	Scheme	Marks
5. (*a*)	 $T_2 = T_1 + mg$	M1
	$\frac{mge}{a} = \frac{mg}{a}(2a - e) + mg$	M1 A1
	$e = \frac{3a}{2} \Rightarrow AE = \frac{5a}{2}$ (*)	A1 A1 cso (5)
(*b*)	$mg + \frac{mg}{a}\left(\frac{1}{2}a - x\right) - \frac{mg}{a}\left(\frac{3}{2}a + x\right) - 2m\sqrt{\frac{g}{a}}\frac{dx}{dt} = m\frac{d^2x}{dt^2}$	M1 A3 (–1eeoo)
	$\Rightarrow \frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = 0$ (*)	A1 (5)
(*c*)	AE: $m^2 + 2km + 2k^2 = 0$	M1
	$m = -k \pm k\text{i}$	A1
	GS: $x = e^{-kt}(A\cos kt + B\sin kt)$	A1 ft
	$t = 0,\ x = \frac{1}{2}a \Rightarrow A = \frac{1}{2}a$	B1
	$\frac{dx}{dt} = ke^{-kt}(A\cos kt + B\sin kt) + e^{-kt}(-kA\sin kt + kB\cos kt)$	M1
	$t = 0,\ \frac{dx}{dt} = 0 \Rightarrow -kA + kB = 0 \Rightarrow B = A = \frac{1}{2}a$	M1
	$x = \frac{1}{2}a\, e^{-kt}(\cos kt + \sin kt)$	A1 (7)
		(17 marks)

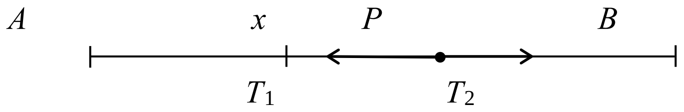
(cso = correct solution only; ft = follow through mark; (*) indicates final line is given on the paper; eeoo = each error or omission)

Question Number	Scheme	Marks
6. (*a*)	No impulse perpendicular to line of centres	
	$\Rightarrow$ velocity perpendicular to line of centres unchanged = $U\cos\alpha$ (*)	B1
	$(\nearrow)$: CLM $U\sin\alpha = v + w$	M1 A1
	NLI $eU\sin\alpha = w - v$	M1 A1
	$\Rightarrow\; v = \tfrac{1}{2}U\sin\alpha(1-e)$	M1 A1 (7)
(*b*)	Component perpendicular to wall = $v\sin\alpha + U\cos\alpha\cos\alpha$	M1
	$= \tfrac{1}{2}U\sin^2\alpha(1-e) + U\cos^2\alpha$	
	$= \tfrac{1}{2}U(\sin^2\alpha - e\sin^2\alpha + 2 - 2\sin^2\alpha)$	M1
	$= \tfrac{1}{2}U[2 - \sin^2\alpha(1+e)]$ (*)	A1
	Component parallel to wall $= U\cos\alpha\sin\alpha - v\cos\alpha$	M1
	$= U\cos\alpha\sin\alpha - \tfrac{1}{2}U\sin\alpha\cos\alpha(1-e)$	A1
	$= \tfrac{1}{2}U\cos\alpha\sin\alpha(1+e)$ (*)	A1 (6)
(*c*)	$e = \tfrac{2}{3}$, $\tan\alpha = \tfrac{3}{4}$	
	Component perpendicular to wall = $\tfrac{1}{2}U(2 - \tfrac{9}{25}\times\tfrac{5}{3}) = \dfrac{7u}{10}$	B1
	Component parallel to wall $= \tfrac{1}{2}U\times\tfrac{4}{5}\times\tfrac{3}{12}\times\tfrac{5}{3} = \dfrac{2u}{5}$	
		
	Distance of A from $X = d\cot\theta = \dfrac{4d}{3}$	B1
	$BX = d\cot\theta$	M1
	$\cot\theta = \dfrac{2u}{5}\times\dfrac{7u}{10} = \dfrac{4}{7}$	A1
	$\therefore$ Total distance $AB = \dfrac{4d}{3} + \dfrac{4d}{7}$	
	$= \dfrac{40d}{21}$	A1 (5)
		(18 marks)

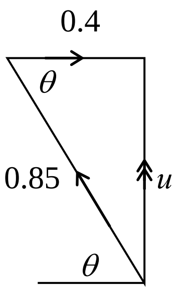
((*) indicates final line is given on the paper)

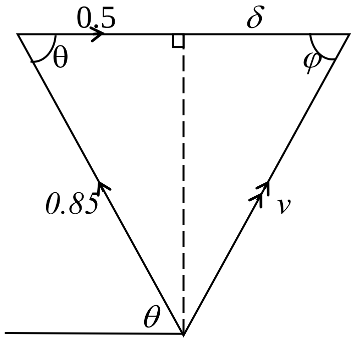
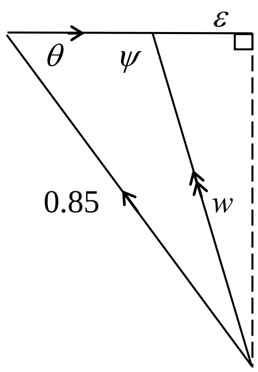
Question Number	Scheme	Marks
1.	N2L $\quad -2v = 3a$	M1
	$-2v = 3v\dfrac{\mathrm{d}v}{\mathrm{d}s}$	A1
	$s = -\frac{3}{2}v\ (+c)$ or $v = -\frac{2}{3}s\ (+c)$ cancelling v and integrating	M1
	$s = 0,\ v = 5 \Rightarrow c = \frac{15}{2}$ or $s = \left[-\frac{3}{2}v\right]_5^2$	M1
	Distance travelled is $4.5\ \mathrm{m}$	A1
		(5 marks)
2.	(a) Before 	
	A: ↑ $\quad u_y = 2.5\sin\alpha = 2.5\times\frac{4}{5} = 2\ \left(\mathrm{ms}^{-1}\right)$ either	M1
	→ $\quad u_x = 2.5\cos\alpha = 2.5\times\frac{3}{5} = 1.5\ \left(\mathrm{ms}^{-1}\right)$ both	A1
	B: ↓ $\quad v_y = 1.3\sin\beta = 1.3\times\frac{12}{13} = 1.2\ \left(\mathrm{ms}^{-1}\right)$ either	M1
	← $\quad v_x = 1.3\cos\beta = 1.3\times\frac{5}{13} = 0.5\ \left(\mathrm{ms}^{-1}\right)$ both	A1 (4)
	(b) After 	
	LM $\quad 2x + w = 3 - 0.5 \quad (= 2.5)$	M1 A1 ft
	NEL $\quad w - x = \frac{1}{2}\times 2 \quad (= 1)$	M1 A1 ft
	Solving $\quad x = 0.5,\ y = 1.5$ $\quad$ M1 solving for either	M1 A1
	Speed of A is $\sqrt{\left(2^2 + 0.5^2\right)} = \sqrt{4.25} \approx 2.1\ \left(\mathrm{m\,s}^{-1}\right)$ M1 either	M1 A1
	Speed of B is $\sqrt{\left(1.2^2 + 1.5^2\right)} = \sqrt{3.69} \approx 1.9\ \left(\mathrm{m\,s}^{-1}\right)$	A1 (9)
		(13 marks)
	Note: *Not* 1 *d.p. loses maximum of one mark*	

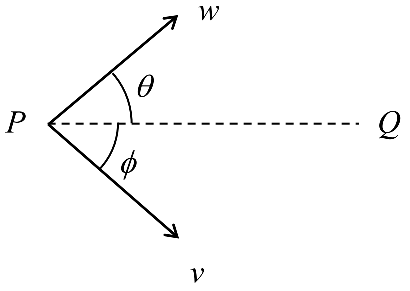
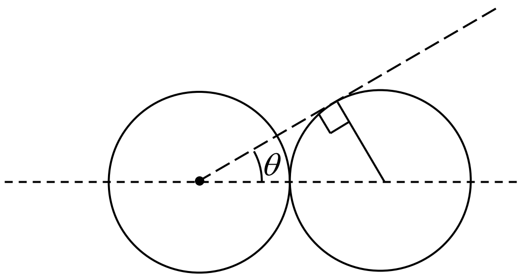
Question Number	Scheme	Marks
3.	(a) $AP = s - AD - DE$	M1
	$= s - L - 2L\sin\theta$	A1 (2)
	(b) $V(\theta) = 2\times 2mg \times L\cos\theta + ...$	B1
	$= \quad ... \quad + mg(2L\cos\theta - AP)$	M1
	$= 4mgL\cos\theta + mg\left(2L\cos\theta + 2L\sin\theta\right)\left(+C\right)$	M1
	$= 2mgL\left(3\cos\theta + \sin\theta\right) +$ constant (*) cso	A1 (4)
	(c) V`(θ) = 2mgL ($-3\sin\theta + \cos\theta$)	M1
	= 0	M1
	$\tan\theta = \frac{1}{3}$	A1
	$\theta \approx 18°$ awrt 18°, 0.32^c	A1 (4)
	(d) $V''(\theta) = 2mgL\left(-3\cos\theta - \sin\theta\right)$	M1 A1
	$\left(V''\left(\arctan\frac{1}{3}\right) = -2\sqrt{10}mgL\right)$	
	V´´(θ) < 0, for any acute θ	M1
	Equilibrium is <u>unstable</u> ft any acute θ	A1 ft (4)
		(14 marks)

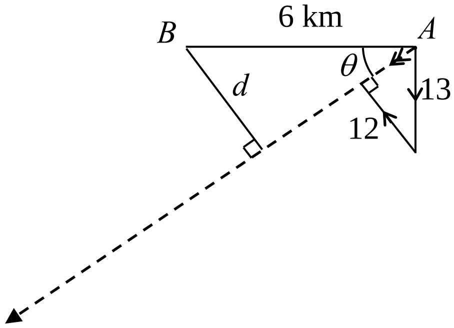
Question Number	Scheme	Marks
4.	(a) 	
	HL $T_1 = \frac{2mk^2L\left(0.5L+x\right)}{L}$ either	M1
	HL $T_2 = \frac{2mk^2L\left(0.5L-x\right)}{L}$ both	A1
	N2L $T_2 - T_1 - 2mk\frac{\mathrm{d}x}{\mathrm{d}t}, = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	M1 A1, A1
	$4mk^2x - 2mk\frac{\mathrm{d}x}{\mathrm{d}t} = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 4k^2x = 0$ * cso	A1 (6)
	(b) $m^2 + 2km + 4m^2 = 0$ ae	M1
	$m = -k \pm k\sqrt{3}\mathrm{i}$	M1
	$x = \mathrm{e}^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$ oe	A1
	$t = 0, x = \frac{L}{2} \Rightarrow A = \frac{L}{2}$	B1
	$\dot{x} = -k\,\mathrm{e}^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$ $+\sqrt{3}k\,\mathrm{e}^{-kt}\left(-A\sin\sqrt{3}kt + B\cos\sqrt{3}kt\right)$	M1
	$t = 0, \dot{x} = 0 \Rightarrow 0 = -kA + \sqrt{3}kB$	M1
	$B = \frac{1}{\sqrt{3}}A = \frac{L}{2\sqrt{3}}$	A1
	$AP = 1.5L + \frac{L}{2\sqrt{3}}\mathrm{e}^{-kt}\left(\sqrt{3}\cos\sqrt{3}kt + \sin\sqrt{3}kt\right)$ oe	A1 (8)
		(14 marks)
	Alternatives forms of the answer are given on the next page	

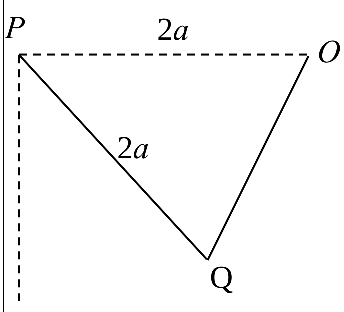
Question Number	Scheme	Marks
4.	(b) *Alternative form of the General Solution* As before $x = Ae^{-kt}\cos(\sqrt{3}kt - \varepsilon)$ $t = 0, x = \frac{L}{2} \Rightarrow \frac{L}{2} = A\cos(-\varepsilon)\ (= A\cos\varepsilon)$ $\dot{x} = -kAe^{-kt}\cos(\sqrt{3}kt - \varepsilon) - \sqrt{3}kAe^{-kt}\sin(\sqrt{3}kt - \varepsilon)$ $t = 0, \dot{x} = 0 \Rightarrow 0 = -kA\cos\varepsilon - \sqrt{3}kA\sin(-\varepsilon)$ Leading to $\tan\varepsilon = \frac{1}{\sqrt{3}} \Rightarrow \varepsilon = \frac{\pi}{6}$ and $A = \frac{L}{\sqrt{3}}$ both $AP = 1.5L + \frac{L}{\sqrt{3}}e^{-kt}\cos\left(\sqrt{3}kt - \frac{\pi}{6}\right)$ *Note: Another possible trig form is* $\sin\left(\sqrt{3}kt + \frac{\pi}{3}\right)$	M1 M1 A1 B1 M1 M1 A1 A1 (8)
5.	(a)Before After  $\rightarrow$ LM $600u = 800x$ $\rightarrow$ NEL $x = eu$ $e = 0.75$	 M1 A1 M1 A1 A1 (5)
	(b) Van N2L $-500 = 800a$ $0^2 = x^2 - 2\times0.625\times45$, $x^2 = 56.25$ $(x = 7.5)$ Car N2L $-300 = 600a$ $0^2 = v^2 - 2\times0.5\times21$, $v^2 = 21$ From (a) NEL $u = \frac{4}{3}\times7.5 = 10$ $V^2 = 10^2 + 21, \Rightarrow V = 11\ (\text{ms}^{-1})$ cao	M1 M1, A1 M1 M1, A1 M1 M1, A1 (9) **(14 marks)**

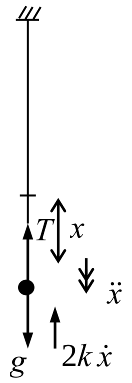
Question Number	Scheme	Marks	
6.			
	(a) Vector ! or ←	M1	
	$\cos\theta = \frac{0.4}{0.85}$	M1	
	$\theta \approx 61.9°$ awrt $62°$	A1	(3)
	(b) $u = \sqrt{(0.85^2 - 0.4^2)}$ or $u = 0.85\sin\theta$	M1	
	$t = \frac{60}{u} = \frac{60}{0.75} = 80 \quad (\text{s})$ cao	M1 A1	(3)
	(c) $\mathbf{v}_{N\text{ rel }W} = -0.4\mathbf{i} \ (+0.75\mathbf{j})$ Allow for $\pm 0.4\mathbf{i}$	M1	
	$\mathbf{v}_N = \mathbf{v}_{N\text{ rel }W} + 0.5\mathbf{i} = 0.1\mathbf{i} + (0.75\mathbf{j})$ $0.1\mathbf{i}$	A1	
	$t = \frac{40}{0.75} = \frac{160}{3}$	M1	
	$\delta = 0.1 \times \frac{160}{3} = \frac{16}{3}$ awrt 5.3	M1 A1	(5)
	(d) As in (c) $\mathbf{v}_N = -0.2\mathbf{i} + 0.75\mathbf{j}$ $\pm 0.2\mathbf{i}$	M1	
	$t = \frac{20}{0.75} = \frac{80}{3}$	M1	
	$\delta = 0.2 \times \frac{80}{3} = \frac{16}{3}$	M1	
	Hence N lands at D cso	A1	(4)
		(15 marks)	
	Notes: 1. *In* (c) *and* (d), *the candidate can take components without using vectors. Mark as vector method.* 2. *After the first line in* (d), *the result is clear by proportion. Allow as long as some explanation given.* 3. $\cos\theta = \frac{8}{17} = 0.4705...$, $\sin\theta = \frac{15}{17} = 0.8823...$ 4. *Alternatives to* (c) *and* (d), *using vector triangles are given on the next page.*		

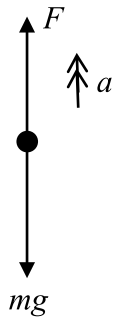
Question Number	Scheme	Marks
6.	*Alternatives to* (c) *and* (d)	
(c)		
	$v^2 = 0.5^2 + 0.85^2 - 2 \times 0.5 \times 0.85 \times \cos\theta$	
	$= 0.5725$ ($v = \frac{\sqrt{229}}{20}$ $\approx 82.4°$)	M1
	$\frac{\sin\varphi}{0.85} = \frac{\sin\theta}{v}$	M1
	$\sin\varphi = \frac{15}{\sqrt{229}}$ (≈ 0.9912; $\varphi \approx 82.4°$)	A1
	$\frac{\delta}{40} = \cot\varphi$; $\delta = 40 \times \frac{2}{5} = \frac{16}{3}$ awrt 5. 3	M1 A1 (5)
(d)	$w^2 = 0.2^2 + 0.85^2 - 2 \times 0.2 \times 0.85 \times \cos\theta$	
		
	$= 0.6025$ $\left(w = \frac{\sqrt{241}}{20} \approx 0.7762...\right)$	M1
	$\frac{\sin\psi}{0.85} = \frac{\sin\theta}{w}$	
	$\sin\psi = \frac{15}{\sqrt{241}}$ (≈ 0.9662; $\psi \approx 104.9°$	M1
	$\psi = 75.1°$ gains M1	
	$\frac{\varepsilon}{20} = \cot(180° - \psi) = \frac{4}{15}$	
	$\varepsilon = \frac{16}{3} = \delta$	M1
	Hence N lands at D cso	A1 (4)
	Note: Exact working is needed for final A1 *but all previous marks in* (c) *and* (d) *may be gained by approximate working*.	

Question Number	Scheme	Marks	
1. (*a*)			
	$v_A = v_{AW} + v_W$		
	$\Rightarrow \begin{pmatrix} k \\ 0 \end{pmatrix} = \begin{pmatrix} v\cos\phi \\ -v\sin\phi \end{pmatrix} + \begin{pmatrix} w\cos\theta \\ w\sin\theta \end{pmatrix}$	M1	
	$\Rightarrow v \sin \phi = w \sin \theta$ *	A1	(2)
(*b*)	$k = v \sin \phi + w \sin \theta$	M1 A1	
	$= \dfrac{v\sqrt{v^2 - w^2 \sin^2\theta}}{v} + w \cos \theta$	M1	
	$= \sqrt{v^2 - w^2\sin^2\theta} + w\cos\theta$	A1	(4)
		(6 marks)	
2.			
	$\sin \theta = \dfrac{a}{2a}$	M1	
	$\Rightarrow \theta = 30°$	A1	
	$v_1 + v_2 = u \cos \theta$	M1 A1	
	$-v_1 + v_2 = eu \cos \theta$	M1 A1	
	$\dfrac{u\sin\theta}{v_1} = \tan(\theta + 30°)$ (or equivalent)	M1 A1	
	Producing an equation in e only	M1	
	$e = \dfrac{1}{3}$	A1 c.s.o	
		(11 marks)	

Question Number	Scheme	Marks
3. (*a*)	 Vector △	M1 A1
	$\cos\theta = \frac{12}{13}$	M1
	($\theta = 22.6°$)	
	Course is 360° – 22.6°	
	= 337° (AWRT)	A1 (4)
(*b*)	$v = \sqrt{13^2 - 12^2} = 5$	
	$t = \dfrac{6\cos\theta}{5} = 1.107$	
	Time is 1.06 p.m.	
(*c*)	$d = 6\sin\theta = 6 \times \frac{5}{13} = 2.31$ km (AWRT 2.3 km)	M1 A1 (2)
		(11 marks)

Question Number	Scheme	Marks
4. (*a*)	 $OQ = 4a\sin\theta$	B1
	$V = (-)\, mga\sin 2\theta;\ + \dfrac{mg}{2\sqrt{3}2a}(4a\sin\theta - a)^2 + C$	B1; M1 A1
	$= -mga\sin 2\theta + \dfrac{mga^2}{4a\sqrt{3}}(16\sin^2\theta - 8\sin\theta + 1) + C$	M1
	$= -mga\sin 2\theta + \dfrac{mga}{4\sqrt{3}}(8(1-\cos 2\theta) - 8\sin\theta) + C$	M1
	i.e. $V = -\dfrac{mga}{\sqrt{3}}(2\cos 2\theta + \sqrt{3}\sin 2\theta + 2\sin\theta) + C$ *	A1 c.s.o (7)
(*b*)	$V'(\theta) = -\dfrac{mga}{\sqrt{3}}(-4\sin 2\theta + 2\sqrt{3}\cos 2\theta + 2\cos\theta)$	M1 A1
	$V'\left(\frac{\pi}{6}\right) = -\dfrac{mga}{\sqrt{3}}\left(-2\sqrt{3} + 2\sqrt{3}\,\frac{1}{2} + 2\,\frac{\sqrt{3}}{2}\right) = 0$	M1 A1 (4)
(*c*)	$V''(\theta) = \dfrac{mga}{\sqrt{3}}(+8\cos 2\theta + 4\sqrt{3}\sin 2\theta + 2\sin\theta)$	M1 A1
	Hence, $V''\left(\frac{\pi}{6}\right) = \dfrac{11mga}{\sqrt{3}} > 0 \therefore$ stable	M1 A1 c.s.o (4)
		(15 marks)

Question Number	Scheme	Marks
5. (*a*)	 R(↓), $mg - 2k\dot{x}m - T = m\ddot{x}$	
	$g - 2k\dot{x} - \dfrac{2ak^2x}{a} = \ddot{x}$	M1
	$\Rightarrow \ddot{x} + 2k\dot{x} + 2k^2x = g$ *	A1 c.s.o (4)
(*b*)	$t = 0, x = a$: $a = D + \dfrac{g}{2k^2} \Rightarrow D = a - \dfrac{g}{2k^2}$	M1 A1
	$\dot{x} = -ke^{-kt}(C\sin kt + D\cos kt) + -ke^{-kt}(C\cos kt - D\sin kt)$	M1 A1
	$t = 0, \dot{x} = 0$: $0 = -kD + kC \Rightarrow C = D$	
	$\Rightarrow C = a - \dfrac{g}{2k^2}$	A1 (5)
(*c*)	$\dot{x} = 0 \Rightarrow$ $C(\sin kt + \cos kt) + C(\cos kt - \sin kt)$	M1
	$\Rightarrow \sin kt = 0$	A1 ft
	$\Rightarrow kt = \pi$	
	$\Rightarrow t = \frac{\pi}{k}$	A1 (3)
(*d*)	When $t = \frac{\pi}{k}$, $x = -De^{-\pi} + \dfrac{g}{2k^2}$	M1
	$\dfrac{g}{2k^2} - e - \pi(a - \dfrac{g}{2k^2})$	A1 ft
	$\Rightarrow xe^{\pi} = \dfrac{g}{2k^2}(e^{\pi} + 1) - a$	
	> 0 (given)	M1
	$\Rightarrow g(e^{\pi} + 1 > 2k^2a$ *	A1 c.s.o (4)
		(16 marks)

Question Number	Scheme	Marks
6. (*a*)	 $F = \frac{kmg}{v}$	B1
	R(↑), $F - mg = ma$	M1
	$\frac{kmg}{v} - mg = mv\frac{dv}{dx}$	M1
	$g(k - v) = v^2\frac{dv}{dx}$ *	A1 (4)
(*b*)	$g\,dx = \frac{v^2}{k-v}dv$	M1
	$\int g\,dx = \int -v - k + \frac{k^2}{k-v}dv$	M1 A1
	$gx = -\frac{v^2}{2} - kv - k^2\ln(k - v) + (c)$	M1 A1
	$x = 0, v = 0$	
	$0 = 0 - 0 - k^2 \ln k + c$	M1
	$c = k^2 \ln k$	
	$gx = -\frac{v^2}{2} - kv - k^2 \ln\left(\frac{k}{k-v}\right)$ *	A1 (7)
(*c*)	Work done by engine = Energy gain	
	$kmgt = \frac{1}{2}mv^2 + mgx$	M1 A1
	$kmgt = mk^2 \ln\left(\frac{k}{k-v}\right) - mkv$	M1
	$\Rightarrow t = \frac{k}{g}\ln\left(\frac{k}{k-v}\right) - \frac{v}{g}$	A1 (5)
		(16 marks)

Edexcel Mechanics M4 – January 2005 **Answers**

1. (a) Impulse on A is in the direction of the line of centres.

Impulse on A $=\Delta(mv)=m(-2\mathbf{i}+5\mathbf{j})-m(\mathbf{i}+2\mathbf{j})=m(-3\mathbf{i}+3\mathbf{j})$.

Therefore direction of line of centres is $(-\mathbf{i}+\mathbf{j})$. A unit vector in this direction is $\dfrac{(-\mathbf{i}+\mathbf{j})}{\sqrt{2}}$.

(b) Let velocity of b after collision be $v_1\mathbf{i}+v_2\mathbf{j}$

Momentum conserved: $m(\mathbf{i}+2\mathbf{j})+5m(-\mathbf{i}+3\mathbf{j})=m(-2\mathbf{i}+5\mathbf{j})+5m(v_1\mathbf{i}+v_2\mathbf{j})$

Cancel m and equate coefficients: $\mathbf{i}$: $-4=-2+5v_1$ $\quad v_1=-\tfrac{2}{5}$

$\mathbf{j}$: $17=5+5v_2$ $\quad v_2=\tfrac{12}{5}$

Velocity of B after collision $=-\tfrac{2}{5}\mathbf{i}+\tfrac{12}{5}\mathbf{j}$.

2. (a) Velocity of wind relative to man $=\mathbf{V}_{WM}=\mathbf{V}_W-\mathbf{V}_M$. $\therefore v(3\mathbf{i}-4\mathbf{j})=\mathbf{V}_W-u\mathbf{j}$

Similarly $w\mathbf{i}=\mathbf{V}_W-\tfrac{1}{5}u(-3\mathbf{i}+4\mathbf{j})$.

Equate the two expressions for $\mathbf{V}_M$ that these produce: $v(3\mathbf{i}-4\mathbf{j})+u\mathbf{j}=w\mathbf{i}+\tfrac{1}{5}u(-3\mathbf{i}+4\mathbf{j})$

Equate coefficients: $\mathbf{i}$: $-3v=w-\tfrac{3}{5}u$

$\mathbf{j}$: $-4v+u=\tfrac{4}{5}u$ $\quad \therefore v=\tfrac{1}{20}u$

(b) $\mathbf{V}_W=\tfrac{1}{20}u(3\mathbf{i}-4\mathbf{j})+u\mathbf{j}=\tfrac{1}{20}u(3\mathbf{i}+16\mathbf{j})$

3. Treat B when $t=0$ as the origin.

$\mathbf{r}_A=12t\mathbf{i}+4\mathbf{j}$. $\quad \mathbf{r}_A=16t\left(\dfrac{\sqrt{3}}{2}\mathbf{i}+\dfrac{1}{2}\mathbf{j}\right)$

$\mathbf{BA}=\mathbf{r}_A-\mathbf{r}_B=\mathbf{i}(12t-8t\sqrt{3})+\mathbf{j}(4-8t)$

Length of $\mathbf{AB}$ $=\sqrt{\left((12t-8t\sqrt{3})^2+(4-8t)^2\right)}=\sqrt{\left((144-192\sqrt{3}+192)t^2+16-64t+64t^2\right)}$

Minimum when derivative of terms inside square root = 0:

$2t(144-192\sqrt{3}+192)-64+128t=0$, $t\approx 0.47$. (Minimum because this is a +ve quadratic.)

Substitute back into length of $\mathbf{AB}$: $|\mathbf{AB}|\approx 0.90$ km.

4. Apply $F = ma$: $m\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{RU}{v} - R$

Separate the variables: $\int_{\frac{u}{4}}^{\frac{u}{2}} \frac{mv\mathrm{d}v}{R(U-v)} = \int_0^T \mathrm{d}t$.

$$\frac{m}{R}\int_{\frac{u}{4}}^{\frac{u}{2}}\left(-1+\frac{U}{(U-v)}\right)\mathrm{d}v = [t]_0^T = T$$

$$\frac{m}{R}\left[-v - U\ln|U-v|\right]_{\frac{u}{4}}^{\frac{u}{2}} = T$$

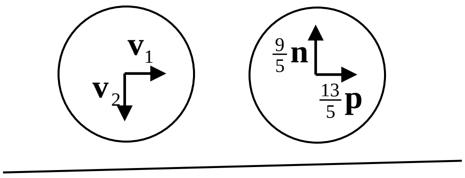
$$T = \frac{m}{R}\left(\left(-\tfrac{1}{2}U - U\ln\left(\tfrac{1}{2}U\right)\right) - \left(-\tfrac{1}{4}U - U\ln\left(\tfrac{3}{4}U\right)\right)\right)$$

$$T = \frac{mU}{R}\left(-\tfrac{1}{4} + \ln\left(\tfrac{3}{2}\right)\right)$$

5. (Note: error in question: $\mathbf{p} = +\tfrac{4}{5}\mathbf{i} + \tfrac{3}{5}\mathbf{j}$.)

(a) $\tfrac{9}{5}\mathbf{n} + \tfrac{13}{5}\mathbf{p} = \tfrac{9}{5}\left(-\tfrac{3}{5}\mathbf{i} + \tfrac{4}{5}\mathbf{j}\right) + \tfrac{13}{5}\left(\tfrac{4}{5}\mathbf{i} + \tfrac{3}{5}\mathbf{j}\right) = \tfrac{25}{25}\mathbf{i} + \tfrac{75}{25}\mathbf{j} = \mathbf{i} + 3\mathbf{j}$

(b) Before After



No impulse parallel to the wall so velocity parallel to wall unchanged: $\mathbf{v}_1 = \tfrac{13}{5}\mathbf{p}$

Newton's law of Restitution perpendicular to the wall: $e\mathbf{v}_2 = -\tfrac{9}{5}\mathbf{n}$

Put in values: $\tfrac{9}{16}\mathbf{v}_2 = -\tfrac{9}{5}\left(-\tfrac{3}{5}\mathbf{i} + \tfrac{4}{5}\mathbf{j}\right)$, $\mathbf{v}_2 = -\tfrac{16}{5}\left(-\tfrac{3}{5}\mathbf{i} + \tfrac{4}{5}\mathbf{j}\right) = \tfrac{39}{25}\mathbf{i} - \tfrac{64}{25}\mathbf{j}$

$\mathbf{v}_1 + \mathbf{v}_2 = \tfrac{13}{5}\left(\tfrac{4}{5}\mathbf{i} + \tfrac{3}{5}\mathbf{j}\right) - \tfrac{16}{5}\left(-\tfrac{3}{5}\mathbf{i} + \tfrac{4}{5}\mathbf{j}\right) = 4\mathbf{i} - \mathbf{i}$

(c) Change in KE $= \tfrac{1}{2}\times\tfrac{1}{2}\times\left(4^2 + 1^2\right) - \tfrac{1}{2}\times\tfrac{1}{2}\times\left(3^2 + 1^2\right) = 1.75$ J

6. (a) Take O as zero p.e.

Mechanical potential energy (mgh) $= -mga\cos 2\theta$

Elastic potential energy $\left(\dfrac{\lambda x^2}{2l}\right) = \frac{1}{2}\times\dfrac{4mg}{\frac{5}{4}a}\left(2a\cos 2\theta - \frac{5}{4}a\right)^2$

Total p.e. $= -mga\left(2\cos^2\theta - 1\right) + \dfrac{8mg}{5a}\left(\dfrac{8a\cos\theta - 5a}{4}\right)^2$

$$= -2mga\cos^2\theta + mga + \frac{mga}{10}\left(8\cos\theta - 5\right)^2$$

$$= \frac{mga}{10}\left(8\cos\theta - 5\right)^2 - 2mga\cos^2\theta + c$$

(change of constant with referral of p.e. to any other zero position.)

(b) Equilibrium when p.e. is max/min so $\dfrac{dE}{d\theta} = 0$

$$\frac{dE}{d\theta} = \frac{mga}{10}\times 16\times\left(8\cos\theta - 5\right)\left(-\sin\theta\right) + 4mga\cos\theta\sin\theta = 0$$

$$mga\sin\theta\left(-\tfrac{64}{5}\cos\theta + 8 + 4\cos\theta\right) = 0$$

$$mga\sin\theta\left(8 - \tfrac{44}{5}\cos\theta\right) = 0$$

$\sin\theta = 0$, $\theta = 0$

or

$\cos\theta = \frac{10}{11}$, $\theta = 24.6°$

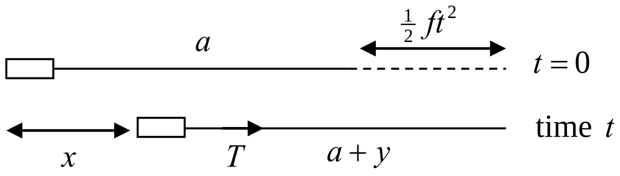
(c) $$\frac{d^2E}{d\theta^2} = mga\sin\theta\left(\tfrac{44}{5}\sin\theta\right) + mga\cos\theta\left(8 - \tfrac{44}{5}\cos\theta\right)$$

When $\theta = 0$, $\dfrac{d^2E}{d\theta^2} = mga\left(8 - \frac{44}{5}\right) = -\frac{4}{5}mga$ which is < 0 so max E so unstable.

When $\theta = 24.6°$, $\dfrac{d^2E}{d\theta^2} = mga\left(\frac{44}{5}\left(1 - \left(\frac{10}{11}\right)^2\right) + \frac{10}{11}\left(8 - \frac{44}{5}\times\frac{10}{11}\right)\right) = \frac{84}{55}mga$

which is > 0 so min E so stable.

7. (a)



At time t:

- the particle has moved x,
- the string is length $(a+y)$,
- the end of the string has moved $\frac{1}{2}ft^2$. $\therefore a+\frac{1}{2}ft^2 = x+a+y$, $\therefore x+y=\frac{1}{2}ft^2$.

(b) $F = ma$ to particle: $T = m\ddot{x}$

$$\frac{man^2}{a}y = m\ddot{x}$$

$$n^2\left(\tfrac{1}{2}ft^2 - x\right) = \ddot{x}$$

$$\ddot{x} + n^2x = \tfrac{1}{2}n^2ft^2$$

(c) $x = 0,\ t = 0\ \therefore 0 = A - \dfrac{f}{n^2},\ A = \dfrac{f}{n^2}$

Differentiating the solution: $\dot{x} = -nA\sin nt + nB\cos nt + ft$ (or get $\ddot{x}$ from original d.e.)
$\dot{x} = 0,\ t = 0\ \therefore 0 = nB,\ B = 0$

(d) Differentiating again: $\ddot{x} = -n^2A\cos nt - n^2B\sin nt + f = -f\cos nt + f$

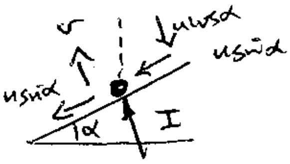
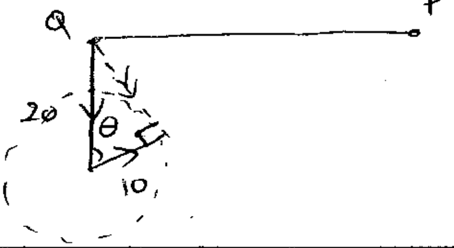
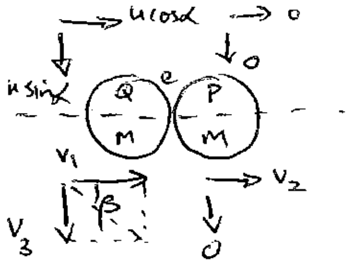
$T = m\ddot{x} = mf(1-\cos nt)$.

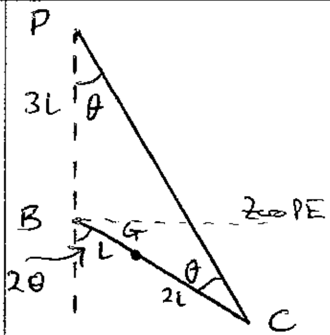
This takes max value of $2mf$ (when $\cos nt = -1$).

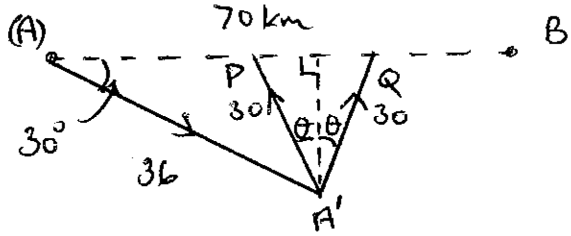
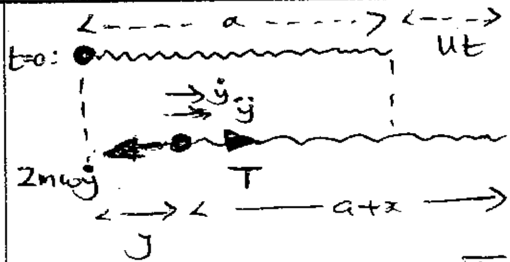
Publication version
04/07/05.

Post standardisation.
MJJ.

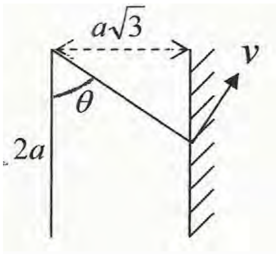
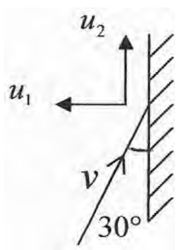
June 2005
6680 Mechanics M4
Mark Scheme

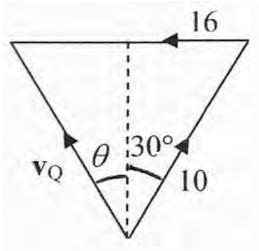
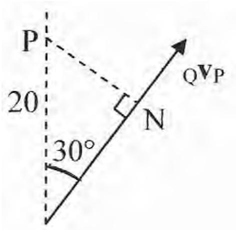
Question Number	Scheme	Marks
1. (a)	 Cpt along plane after impact $= 10\sin\alpha$ $= 10 \times 3/5$ $= 6$	B1
	$v = e \times 10\cos\alpha$ $\left(= \frac{1}{2} \times 10 \times \frac{4}{5} = 4\right)$	M1 A1
	Speed $= \sqrt{4^2+6^2} = \underline{7.21 \text{ ms}^{-1} \text{ (3SF)}}$	M1 A1 (5)
(b)	$I = \frac{1}{2}(4 - -8) = \underline{6\text{Ns}}$	M1 A1 (2) (7)
2.	Fix P  Vector Δ $\cos\theta = \frac{10}{20}$ $\Rightarrow \underline{\theta = 060°}$	M1 A1 M1 A1 A1 (5)
3.	 $v_3 = u\sin\alpha$	B1
	CLM: $v_1 + v_2 = u\cos\alpha$	M1 A1
	NIL: $-v_1 + v_2 = eu\cos\alpha$	M1 A1
	$\frac{v_3}{v_1} = \tan\beta$	M1 A1
	elim v_2	M1
	elim v_3	M1
	elim u	M1
	$\Rightarrow \underline{\tan\beta(1-e) = 2\tan\alpha}$ *	A1 (11)

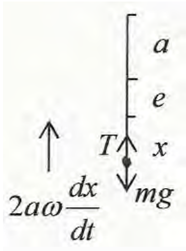
Question Number	Scheme	Marks
4 (a)	For constant speed, $F - kV^2 = 0$	M1
	$\Rightarrow V = \sqrt{F/k}$ *	A1 (2)
(b)	$F - kv^2 = Ma$	M1 A1
	$\Rightarrow F - kv^2 = Mv\frac{dv}{dx}$	M1
	$\int dx = M\int \frac{v}{F-kv^2}\,dv$	M1
	$x = -\frac{M}{2k}\ln(F-kv^2)\ (+c)$	A1
	$x=0, v=0 \Rightarrow c = \frac{M}{2k}\ln F$	M1 A1
	$x = \frac{M}{2k}\{\ln F - \ln(F-kv^2)\}$	
	$X = \frac{M}{2k}\ln\left(\frac{F}{F-k.\frac{F}{4k}}\right)$	M1
	$= \frac{M}{2k}\ln\frac{4}{3}$ *	A1 (9)
		(11)
5. (a)	 $GPE = -mgl\cos 2\theta$	B1
	$EPE = \frac{mg}{6}\frac{(6l\cos\theta - l)^2}{2l}$	M1
	$= \frac{mg}{12l}(36l^2\cos^2\theta - 12l^2\cos\theta + l^2)$	M1
	$= mgl(3\cos^2\theta - \cos\theta) + C$	
	$V = -mgl(2\cos^2\theta - 1) + mgl(3\cos^2\theta - \cos\theta) + c$	M1
		M1 use of cos2θ=...
	$= mgl(\cos^2\theta - \cos\theta) + c'$ *	A1 (6)
(b)	$\frac{dV}{d\theta} = mgl(-2\cos\theta\sin\theta + \sin\theta) = 0$	M1 A1; M1
	$\sin\theta(-2\cos\theta + 1) = 0$	
	$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$	M1
	$\theta = 0$ or $\theta = \pm\pi/3$	A1 A1
		(6)
		(12)

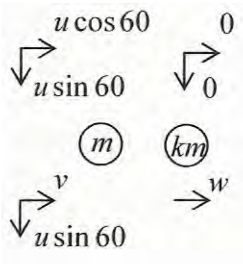
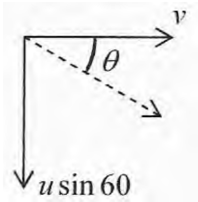
6. (a)	 Minimum speed for interception $= 36\sin 30° = 18$	M1 A1
	$\cos\theta = \frac{18}{30} \left(= \frac{3}{5}\right)$	M1 A1
	$\Rightarrow \tan\theta = 4/3$	A1
	Explanation	M1
		A1 cso (7)
(b)	$AQ = 36\cos 30° + 30\sin\theta$	M1 A2
	$(18\sqrt{3} + 24)$	
	$\text{Time} = \frac{70}{(18\sqrt{3}+24)} = 1.27 \text{ hrs.}$	M1 A1 (5)
		(12)
7. (a)	 $a + ut = y + (a + x)$	M1
	$ut = y + x$ *	A1 (2)
(b)	For particle, $R(\rightarrow)$, $T - 2m\omega\dot{y} = m\ddot{y}$	M1 A1
	$T = \frac{5ma\omega^2 x}{a}$	M1
	$u = \dot{y} + \dot{x}$; $0 = \ddot{y} + \ddot{x}$	B1; B1
	$5m\omega^2 x - 2m\omega(u - \dot{x}) = m(-\ddot{x})$	M1
	$\Rightarrow \ddot{x} + 2\omega\dot{x} + 5\omega^2 x = 2\omega u$ *	A1 (7)
(c)	AE: $u^2 + 2\omega u + 5\omega^2 = 0 \Rightarrow (u+\omega)^2 = -4\omega^2$	B1
	$\Rightarrow u = -\omega \pm 2i\omega$	M1
	CF : $x = e^{-\omega t}(A\cos 2\omega t + B\sin 2\omega t)$	B1
	PI : $x = \frac{2\omega u}{5\omega^2} = \frac{2u}{5\omega}$	
	GS : $x = e^{-\omega t}(A\cos 2\omega t + B\sin 2\omega t) + \frac{2u}{5\omega}$	B1
	$x = 0, t = 0$: $0 = A + \frac{2u}{5\omega} \Rightarrow A = -\frac{2u}{5\omega}$	M1 A1
	$\dot{x} = -\omega e^{-\omega t}(A\cos 2\omega t + B\sin 2\omega t) + e^{-\omega t}(-2\omega A\sin 2\omega t + 2\omega B\cos 2\omega t)$	M1
	$t = 0, \dot{y} = 0 \Rightarrow \dot{x} = u$	
	$u = -\omega A + 2\omega B \Rightarrow B = \frac{3u}{10\omega}$	
	$x = e^{-\omega t}\left(\frac{3u}{10\omega}\sin 2\omega t - \frac{2u}{5\omega}\cos 2\omega t\right) + \frac{2u}{5\omega}$	A1 (8)
		(17)

MJJ. 4.7.05

Question Number	Scheme	Marks
1.	(a) $\frac{1}{2}\frac{dv}{dt} = \frac{1}{2}g - 2v$	M1
	$\Rightarrow 5\frac{dv}{dt} = 49 - 20v$ (*)	A1 (2)
	(b) $\int \frac{5dv}{49-20v} = \int dt$ (separate variables)	M1
	$\frac{-5}{20}\ln(49-20v) = t$ (+c)	A1
	$t = 0, v = 0 \Rightarrow c = -\frac{1}{4}\ln 49$ (attempt to get c)	M1
	$t = \frac{1}{4}\ln\left(\frac{49}{49-20v}\right)$	
	$t = 1 : 1 = \frac{1}{4}\ln(\frac{49}{49-20v})$ (correct use of logs/exp)	M1
	$\rightarrow v \approx 2.41ms^{-1}$ or $2.4ms^{-1}$	A1 (5)
		Total 7 marks
2.	(a) Energy: $\frac{1}{2}m\left(\frac{37ga}{5} - v^2\right) = mg.2a(1-\cos\theta)$ 	M1 A1
	Using $\theta = \frac{\pi}{3}$ & solve: $\rightarrow v = \sqrt{\frac{27ga}{5}}$ (*)	M1 A1 (4)
	(b) Impact: $u_1 = ev\sin 30$ 	M1 A1
	KE loss $= \frac{1}{2}m\left(v^2\sin^2 30 - e^2v^2\sin^2 30\right)$	
	$\left[+\frac{1}{2}mv^2\cos^2 30 - \frac{1}{2}mu_2{}^2\right] = \frac{3mga}{5}$	M1 A1
	[Using $u_2 = v\cos 30$ if necessary &] reducing to equation in (m, g, a) e alone	
	$\frac{3mga}{5} = \frac{1}{2}m.\frac{27ga}{5}.\frac{1}{4}(1-e^2)$	A1
	Solve for e: $\rightarrow e = \frac{1}{3}$	M1 A1 (7)
		Total 11 marks

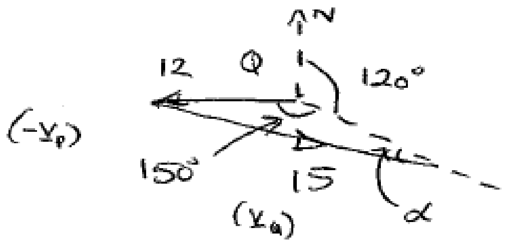
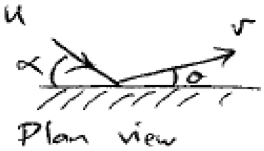
Question Number	Scheme	Marks
3.	(a) (i) $\mathbf{v}_Q = {}_Q\mathbf{v}_P + \mathbf{v}_P$	
	$\lvert\mathbf{v}_Q\rvert^2 = (10\cos 30)^2 + (16 - 10\sin 30)^2$	M1 A1
	$= 75 + 121$	
	$\Rightarrow \underline{\lvert\mathbf{v}_Q\rvert = 14 ms^{-1}}$	A1
		
	(ii) $\tan\theta = \dfrac{16 - \sin 30}{10\cos 30}$ (o.e.)	M1
	$\theta \approx 51.8°$, $\Rightarrow$ bearing $\underline{308°}$ (nearest degree)	A1, A1
		(6)
	(b) At nearest approach: $PN = 20\sin 30$	M1 A1
	$= \underline{10\ km}$	A1
		(3)
	(c) $Time = \dfrac{20\cos 30}{10} \approx 1.732\ hrs$	M1 A1
	$\Rightarrow \underline{Time \approx 4.44\ pm}$ (AWRT)	A1
		(3)
		Total 12 marks
	<u>Alternatives</u>	
	(a) Use of cosine rule in velocity vector triangle.	
	(b) & (c) Use of scalar product of relative velocity and relative position or differentiating magnitude of relative position vector squared to find the minimum distance and time at which it occurs.	

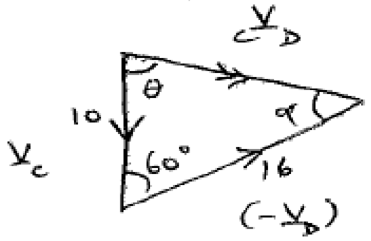
Question Number	Scheme	Marks
4.	(a) R(↓) $m\frac{d^2x}{dt^2} = mg - T - 2m\omega\frac{dx}{dt}$ (4 terms)	M1 A1
	$m\frac{d^2x}{dt^2} = mg - \frac{2m\omega^2 a}{a}(e+x) - 2m\omega\frac{dx}{dt}$	↓ M1
	$\rightarrow \frac{d^2x}{dx^2} + 2\omega\frac{dx}{dt} + 2\omega^2 x = 0$ (*)	↓ M1 A1
		(5)
	(b) $x = e^{-\omega t}(A\cos\omega t + b\sin\omega t)$	
	$t = 0, x = 0 \Rightarrow \underline{A = 0}$	B1
	$\frac{dx}{dt} = -\omega e^{-\omega t}.B\sin\omega t + e^{-\omega t}.B\omega\cos\omega t$ (use of product rule)	M1
	$t - 0, \frac{dx}{dt} = U : U = B\omega \Rightarrow \underline{B = \frac{U}{\omega}}$	M1 A1
		(4)
	(c) $\frac{dx}{dt} = -Ue^{-\omega t}\sin\omega t + Ue^{-\omega t}\cos\omega t = 0$	M1
	$\Rightarrow \tan\omega t = 1$ (solve for $\tan\omega t$)	M1
	$\Rightarrow \underline{t = \frac{\pi}{4\omega}}$	A1
		(3)
		Total 12 marks

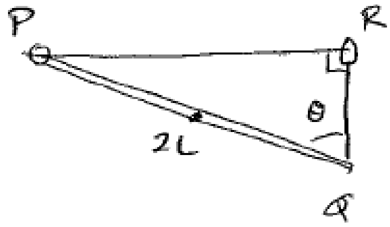
Question Number	Scheme	Marks
5.	(a) $CLM(\leftrightarrow): mu\cos 60 = mv + kmw$	M1 A1
	$NLI: \frac{1}{2}u\cos 60 = w - v$	M1 A1
	Solve for w: $(1+k)w = \frac{1}{2}u\left(1+\frac{1}{2}\right)$	M1
	$\Rightarrow w = \frac{3u}{4(k+1)}$ (*)	A1 (6)
		
	(b) Solve for v $\rightarrow v = \frac{u(2-k)}{4(k+1)}$	M1 A1
	$\tan\theta = 2\sqrt{3} = \frac{u\sin 60}{v}$	M1 A1
	$= \frac{u\sqrt{3}}{2}.\frac{4(k+1)}{u(2-k)}$	
		
	Solve k: $\rightarrow k = \frac{1}{2}$	M1 A1 (6)
	(c) $k = \frac{1}{2} \Rightarrow v = \frac{u}{4}, w = \frac{u}{2}$	B1
	KE loss $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m.\frac{u^2}{16} + \frac{1}{2}m.\frac{3u^2}{4} + \frac{1}{2}.\frac{1}{2}m.\frac{u^2}{4}\right)$	M1 A1
	$= \frac{1}{2}mu^2\left(1 - \frac{1}{16} - \frac{3}{4} - \frac{1}{8}\right)$	
	$= \frac{1}{32}mu^2$	A1 (4)
		Total 16 marks

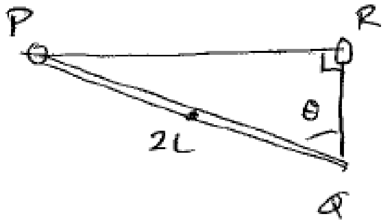
Question Number	Scheme	Marks
6.	(a) PE of R $= -\sqrt{2}mga\cos 2\theta$ (+c) (1)	B1
	PE of LH mass $= -\frac{3}{2}mg(2a - 2a\sin(45+\theta))$ (+c) (2)	M1 A1
	PE of RH mass $= -\frac{3}{2}mg(2a - 2a\sin(45-\theta))$ (+c) (3)	A1
	V = (1) + (2) + (3) (in terms of θ etc.)	M1
	$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mg\left[4a - a\sqrt{2}(\cos\theta + \sin\theta + \cos\theta - \sin\theta)\right]$	M1
	$= -\sqrt{2}mga\cos 2\theta - \frac{3}{2}mga\left(-2\sqrt{2}\cos\theta + 4\right)$	
	$= \sqrt{2}mga(3\cos\theta - \cos 2\theta)$ + constant (*)	A1 (7)
	(b) $\frac{dV}{d\theta} = \sqrt{2}mga(-3\sin\theta + 2\sin 2\theta)$	M1 A1
	$\frac{dV}{d\theta} = 0 \Rightarrow 2\sin 2\theta - 3\sin\theta = 0$	M1
	$\Rightarrow \sin\theta(4\cos\theta - 3) = 0$	M1
	$\Rightarrow \theta = 0,$ or $\theta = \pm\arccos\frac{3}{4}(= \pm 0.723)$	A1, A1 (6)
	(c) $\frac{d^2V}{d\theta^2} = \sqrt{2}mga(-3\cos\theta + 4\cos 2\theta)$	M1 A1
	$\cos\theta = \frac{3}{4}: \frac{d^2V}{d\theta^2} = \sqrt{2}mga\left(-3.\frac{3}{4} + 4\left(2.\frac{9}{16} - 1\right)\right)$	M1
	$= \sqrt{2}mga\left(-\frac{9}{4} + \frac{1}{2}\right)$	
	<0 $\therefore$ Unstable	A1 (4)
		Total 17 marks

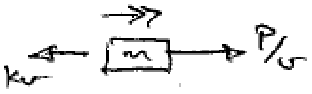
June 2006
6680 Mechanics M4
Mark Scheme

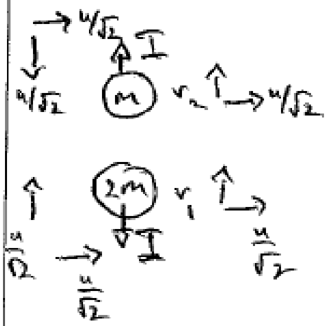
Question Number	Scheme	Marks
1.		M1
	$\frac{\sin\alpha}{12} = \frac{\sin 150°}{15}$	M1 A1
	$\Rightarrow \sin\alpha = \frac{6}{15}$	
	$\Rightarrow \alpha = 23.6°$	A1
	∴ Course is 096 (.4°)	A1 (5)
2.		
	(→) $u\cos\alpha = v\cos\theta$	M1 A1
	(↑) $eu\sin\alpha = v\sin\theta$	M1 A1
	$\Rightarrow v^2 = u^2(\cos^2\alpha + e^2\sin^2\alpha)$	M1
	$\Rightarrow KE = \frac{1}{2}mu^2(\cos^2\alpha + e^2\sin^2\alpha)$	A1 (6)

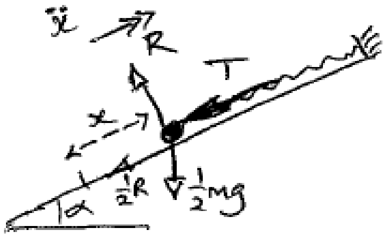
Question Number	Scheme	Marks
3. (a)	 $\lvert \underset{C D}{v} \rvert^2 = 10^2 + 16^2 - 2 \times 10 \times 16 \cos 60°$ $= 196$	M1 A1
	$\lvert \underset{C D}{v} \rvert = 14 \text{ ms}^{-1}$ *	A1 (3)
(b)	α is acute (opposite shortest side)	
	$\frac{\sin\alpha}{10} = \frac{\sin 60°}{14}$	M1
	$\Rightarrow \alpha = 38.213°...$	A1
	 (i) $DN = 4000 \sin 8.213$	M1
	$\simeq 571 \text{ m}$ $\left(\frac{4000}{7}\right)$	A1
	(ii) $t = \frac{4000 \cos 8.213°}{14}$ secs.	M1 A1
	$\simeq 282.78...$ secs.	
	Time is 2.05 pm (nearest minute)	A1 (7)
		(10)

Question number	Scheme	Marks
4.(a)	 PE of rod $= -mgl\cos\theta$	B1
	EPE of string $= \frac{kmg}{2l}(2l\cos\theta - l)^2$	M1 A1
	Total PE of system, $V = -mgl\cos\theta + \frac{kmgl}{2}(2\cos\theta-1)^2 + c$	M1
	$= -mgl\cos\theta + \frac{kmgl}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgl(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= mgl[2k\cos^2\theta - (2k+1)\cos\theta] + c'$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgl(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgl\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\frac{1}{2} < k$ *	A1 (5)
		(12)

Question number	Scheme	Marks
4.(a)	 PE of rod $= -mgl\cos\theta$	B1
	EPE of string $= \frac{kmg}{2l}(2l\cos\theta - l)^2$	M1 A1
	Total PE of system, $V = -mgl\cos\theta + \frac{kmgl}{2}(2\cos\theta-1)^2 + c$	M1
	$= -mgl\cos\theta + \frac{kmgl}{2}(4\cos^2\theta - 4\cos\theta + 1) + c$	M1 A1
	$= mgl(-\cos\theta + 2k\cos^2\theta - 2k\cos\theta) + c'$	
	$= mgl[2k\cos^2\theta - (2k+1)\cos\theta] + c'$ *	A1 (7)
(b)	$\frac{dV}{d\theta} = mgl(-4k\cos\theta\sin\theta + (2k+1)\sin\theta)$	M1 A1
	At equil^m, $mgl\sin\theta(-4k\cos\theta + (2k+1)) = 0$	M1
	$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{2k+1}{4k}$	
	$\Rightarrow \theta = 0$ $(\theta > 0)$ $\frac{2k+1}{4k} < 1$	M1
	$2k+1 < 4k$	
	$1 < 2k$	
	$\frac{1}{2} < k$ *	A1 (5)
		(12)

Question number	Scheme	Marks
5.(a)	 $\frac{P}{v}$	B1
	$(\rightarrow)$: $\frac{P}{v} - kv = m\frac{dv}{dt}$	M1
	$\Rightarrow P = mv\frac{dv}{dt} + kv^2$ *	A1 (3)
(b)	$\int_0^T dt = \int_U^{2U} \frac{mv\,dv}{P-kv^2}$ $\left(U = \frac{1}{3}\sqrt{\frac{P}{k}}\right)$	M1 A1
	$\Rightarrow T = \frac{-m}{2k}\left[\ln(P-kv^2)\right]_U^{2U}$	A2
	$= \frac{m}{2k}\left\{\ln\left(P - \frac{k}{9}\cdot\frac{P}{k}\right) - \ln\left(P - \frac{4k}{9}\frac{P}{k}\right)\right\}$	M1 A1
	$= \frac{m}{2k}\left\{\ln\frac{8P}{9} - \ln\frac{5P}{9}\right\}$	
	$= \frac{m}{2k}\ln\left(\frac{8P}{9}\times\frac{9}{5P}\right)$	M1
	$= \frac{m}{2k}\ln\frac{8}{5}$	A1 (8)
		(11)

Question number	Scheme	Marks
6. (a)		
	Form: $I = m\left(v_2 + \frac{u}{\sqrt{2}}\right)$	M1 A1
	CLM(↑): $2m\frac{u}{\sqrt{2}} - \frac{mu}{\sqrt{2}} = 2mv_1 + mv_2$	
	$\frac{u}{\sqrt{2}} = 2v_1 + v_2$ ①	M1 A1
	NIL: $e\frac{2u}{\sqrt{2}} = \frac{u}{\sqrt{2}} = -v_1 + v_2$ ②	M1 A1
	$\Rightarrow \frac{3u}{\sqrt{2}} = 3v_2$	M1 A1
	$\Rightarrow I = m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)$	
	$= \underline{\underline{mu\sqrt{2}}}$	A1 (9)
(b)	$v_2 - v_1 = \frac{u}{\sqrt{2}}$ (Separation speed)	M1
	time to wall $= \frac{d}{u/\sqrt{2}} = \frac{d\sqrt{2}}{u}$	M1 A1
	∴ Separation $= \frac{d\sqrt{2}}{u} \times \frac{u}{\sqrt{2}} = \underline{\underline{d}}$	M1 A1
		(5)
		(14)

Question number	Scheme	Marks
7.(a)		
	$F = \frac{1}{2}R$	M1
	$R = mg\cos\alpha$	B1
	$T = \frac{4mgx}{L}$	B1
	$(\nearrow): \quad -F - mg\sin\alpha - T = m\ddot{x}$	M1 A1
	$-\frac{1}{2}\cdot\frac{4}{5}mg - \frac{3}{5}mg - \frac{4mgx}{L} = m\ddot{x}$	
	$\Rightarrow \frac{d^2x}{dt^2} + 4\omega^2 x = -g$ *	A1
	$(\omega = \sqrt{g/L})$	(6)
(b)	$m^2 + 4\omega^2 = 0 \Rightarrow m = \pm 2\omega i$	
	C.F. is $x = A\sin 2\omega t + B\cos 2\omega t$	M1
	P.I. is $x = \frac{-g}{4\omega^2} = -L/4$	B1
	G.S. is $x = A\sin 2\omega t + B\cos 2\omega t - \frac{L}{4}$	B1
	$t=0, x=0: \quad B = L/4$	M1 A1
	$\dot{x} = 2\omega A\cos 2\omega t - 2\omega B\sin 2\omega t$	
	$t=0, \dot{x} = \frac{1}{2}\sqrt{gL}: \quad \frac{\sqrt{gL}}{2} = 2\sqrt{\frac{g}{L}}A \Rightarrow A = L/4$	M1
	$\Rightarrow x = \frac{L}{4}(\sin 2\omega t + \cos 2\omega t - 1)$	A1 (7)
(c)	$\dot{x} = 0 \Rightarrow 2\omega A\cos 2\omega t - 2\omega B\sin 2\omega t = 0$	M1
	$\Rightarrow \tan 2\omega t = \frac{A}{B} = 1$	
	$\Rightarrow 2\omega t = \pi/4$ (first value)	A1
	$\Rightarrow x = \frac{L}{4}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right)$	M1
	$= \frac{L}{4}(\sqrt{2} - 1)$	A1 (4)
		(17)

Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6680/01)

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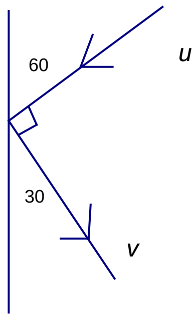
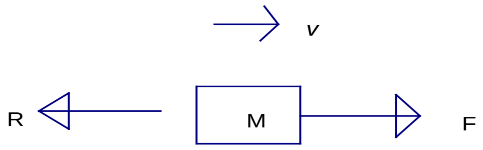
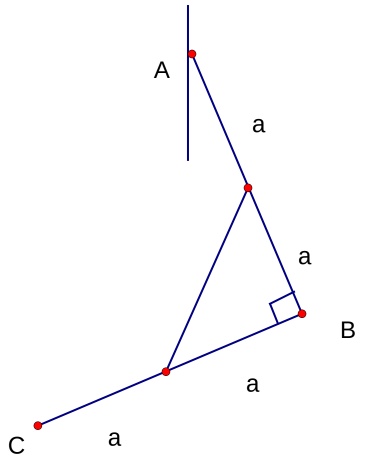
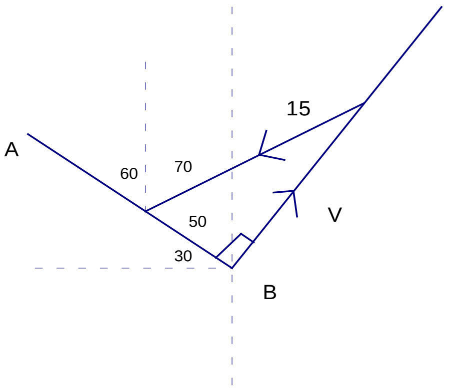
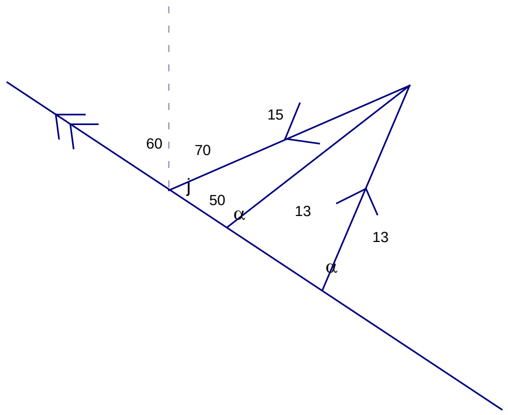
June 2007
6680 Mechanics M4
Mark Scheme

General:
For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
Omission of *g* from a resolution is an accuracy error, not a method error.
Omission of mass from a resolution is a method error.
Omission of a length from a moments equation is a method error.
Where there is only one method mark for a question or part of a question, this is for a *complete* method.
Omission of units is not (usually) counted as an error.

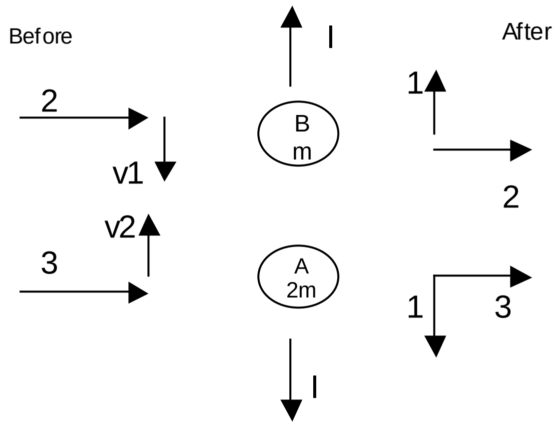
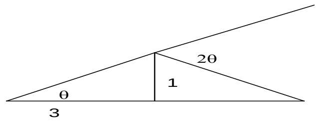
Question Number	Scheme	Marks
1(a)		
	$u\cos 60^0 = v\cos 30^0$	M1A1
	$u = v\sqrt{3}$	A1
	KE lost = $\frac{1}{2}m(u^2 - v^2)$	M1
	Fraction of KE lost = $1 - \left(\frac{v}{u}\right)^2$	DM1
	$= 1 - \frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7	A1
	or $\frac{3}{4}(1 - e^2)$	(6)
(b)	$e = \frac{v\sin 30°}{u\sin 60°}$	M1A1
	$= \frac{v}{u} \cdot \frac{1}{\sqrt{3}}$	DM1
	$= \frac{1}{3}$	A1 (4)
a)	M1 Resolve parallel to the wall *Alt: reasonable attempt at equation connecting two variables* A1 Correct as above or equivalent *equation correct* A1 *u* in terms of *v* or v.v. - not necessarily simplified. *or ration of the two variables correct* M1 expression for KE lost DM1 expression in one variable for fraction of KE lost – could be *u/v* as above A1 cao	*The first three marks can be awarded in (b) if not seen in (a)*
b)	M1 Use NIL perpendicular to the wall and form equation in *e* A1 Correct unsimplified expression as above or $eu\sin 60^o = v\sin 30^o$ or equivalent DM1 Substitute values for trig functions or use relationship from (a) and rearrange to e = ….. A1 cao accept decimals to at least 3sf	*The first two marks can be awarded in (a)*

2(a) (b)	 $F=\frac{Ru}{v}$ R(→), $\frac{Ru}{v}-R=M\frac{dv}{dt}$ $R(u-v)=Mv\frac{dv}{dt}$ * $\int_0^T dt=\frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U}\frac{v\,dv}{u-v}$ $\Rightarrow T=\frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U} -1+\frac{u}{u-v}dv$ $=\frac{M}{R}\left[-v-u\ln(u-v)\right]_{\frac{1}{4}U}^{\frac{1}{3}U}$ $=\frac{M}{R}\left[-\frac{u}{3}-u\ln\left(\frac{2u}{3}\right)+\frac{u}{4}+u\ln\left(\frac{3u}{4}\right)\right]$ $\left(C=-\frac{Mu}{R}\left(\ln\frac{3u}{4}+\frac{1}{4}\right)\right)$ $=\frac{Mu}{R}\left(-\frac{1}{12}+\ln\frac{9}{8}\right)$ Hence $k=\ln\frac{9}{8}-\frac{1}{12}$	B1 M1 A1 (3) M1A1 DM1 A1 M1 M1 A1 (7)
a) b)	B1 Correct expression involving the driving force. M1 Use of F = ma to form a differential equation. Condone sign errors. a must be expressed as a derivative, but could be any valid form. A1 Rearrange to **given form**. M1 Separate the variables A1 Separation correct (limits not necessarily seen at this stage) DM1 Attempt a complete integration process A1 Integration correct M1 Correct use of both limits – substitute and subtract. Condone wrong order. M1 Simplify to find k from an expression involving a logarithm A1 Answer as given, or exact equivalent. Need to see k = lnA + B	

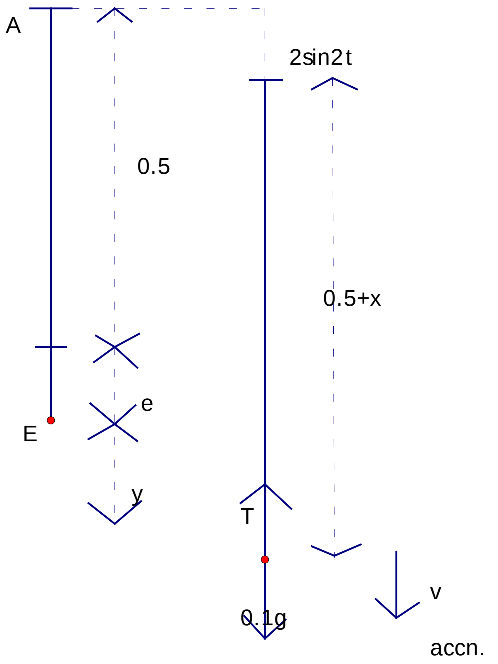
Question Number	Scheme	Marks
3. (a)	 $V = -mga\cos\theta - mg(2a\cos\theta + a\sin\theta)$ $= -mga(3\cos\theta + \sin\theta) \quad (+const)$ *	M1A1A1 A1 (4)
(b)	$\frac{dV}{d\theta} = -mga(-3\sin\theta + \cos\theta)$ $= 0 \Rightarrow \tan\theta = \frac{1}{3}$ $\Rightarrow$ θ = 0.32(1)^c or 18.4^o accept awrt	M1A1 M1 A1 (4)
(c)	$\frac{d^2V}{d\theta^2} = -mga(-3\cos\theta - \sin\theta)$ $= mga(3\cos\theta + \sin\theta)$ Hence, when θ = 0.32^c, $\frac{d^2V}{d\theta^2} > 0$ i.e. stable	M1A1 M1 A1 (4)
a)	M1 Expression for the potential energy of the two rods. Condone trig errors. Condone sign errors. BC term in two parts A1 correct expression for AB A1 correct expression for BC A1 Answer **<u>as given</u>** .	
b)	M1 Attempt to differentiate V. Condone errors in signs and in constants. A1 Derivative correct M1 Set derivative = 0 and rearrange to a single trig function in θ A1 Solve for θ or M1A1 find the position of the center of mass M1A1 form and solve trig equation for θ	
c)	M1 Differentiate to obtain the second derivative A1 Derivative correct M1 Determine the sign of the second derivative A1 Correct conclusion. cso Or: M1 Find the value of $\frac{dV}{d\theta}$ on both sides of the minimum point A1 signs correct M1 Use the results to determine the nature of the turning point A1 Correct conclusion, cso.	*These 4 marks are dependent on the use of derivatives*

4 (a)	 Fix A $v_{\min} = 15\sin 50°$ $=11.5$ km h^{-1} (3 s.f.) or: triangle without the right angle identified and $\frac{15}{\sin\theta} = \frac{v_B}{\sin 50}$ $\Rightarrow v_B = \frac{15\sin 50}{\sin\theta}$ minimum value $\Rightarrow \theta = 90$ for M1 As above for A1A1	M1A1 A1 (3)
(b)	 Ambiguous Sine Rule: 2 possible solutions for α	B1B1 (2)
(c)	$\frac{\sin\alpha}{15} = \frac{\sin 50}{13}$	M1A1
	α = 62,1º (or 118º) (smaller value gives larger relative velocity)	A1
	$\Rightarrow$ either $v = 13\cos 62.1 + 15\cos 50 = 15.72 kmh^{-1}$	M1A1
	Or $v^2 = 15^2 + 13^2 - 390\cos 67.9 = 247.27$ $v = 15.7 kmh^{-1}$	M1 A1
	Time = $\frac{20}{their 15.72.....}$ = 1.272…… hrs	M1 A1
	Earliest time is 13.16hrs or 13.17 hrs accept 1.16 (pm) or 1.17 (pm)	A1 (8)

a)	M1 Velocity of B relative to A is in the direction of the line joining AB. Minimum V requires a right angled triangle. Convincing attempt to find the correct side. A1 15 x sin(their 50^0) A1 Q specifies 3sf, so 11.5 only	
b)	B1B1 Convincing argument B1B0 Argument with some merit	
c)	M1 Use of Sine Rule A1 Correct expression A1 (2 possible values,) pick the correct value. M1 Use trig. to form an equation in v A1 correct equation M1 $time = \frac{distance}{speed}$ A1ft correct expression with their v (not necessarily evaluated) A1 correct time in hours & minutes Or: M1 Use of cosine rule A1 $13^2 = 15^2 + v^2 - 2 \times 15 \times v \times \cos 50$ A1 (Award after the next two marks) 15.72 or awrt 15.72 M1 Attempt to solve the equation for v A1 $\frac{30\cos 50 \pm \sqrt{(30\cos 50)^2 - 4 \times 56}}{2}$ (15.72 or 3.562) Finish as above	

5. (a)		
	CLM: $2v_2 - v_1 = 1 - 2 = -1$	M1A1
	NIL: $1 + 1 = \frac{1}{2}(v_1 + v_2)$	M1A1
	$\therefore$ $v_2 = 1$, $v_1 = 3$ *Dependent on both M's above*	DM1
		A1
	Horizontal components unchanged (i.e. 2 & 3) *Independent of all other marks*	A1
	$\mathbf{v}_A = 3\mathbf{i} + \mathbf{j}$; $\mathbf{v}_B = 2\mathbf{i} - 3\mathbf{j}$	(7)
(b)		M1A1
	For B: I = m(1-(-3)) = 4m	(2)
(c)	(Or For A: -I = 2m(-1 – 1) $\therefore$ I = 4m)	
	$\begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}3\\-1\end{pmatrix} = \sqrt{3^2+1^2}.\sqrt{3^2+(-1)^2}\cos\theta$	M1A1
	$\Rightarrow 8 = 10\cos\theta$	M1
	$\theta = 37^{o}$	A1
		(4)
	Alternative:	
	 M1	
	where $\tan\theta = \frac{1}{3}$ A1	
	required angle is 2θ M1A1	

a)	M1 Conservation of momentum along the line of centres. Condone sign errors A1 equation correct M1 Impact law along the line of centres. *e* must be used correctly, but condone sign errors. A1 equation correct. The signs need to be consistent between the two equations M1 Solve the simultaneous equations for their v_1 and v_2. A1 **i** components correct – independent mark A1 $\mathbf{v}_A$ & $\mathbf{v}_B$ correct	
b)	M1 Impulse = change in momentum for one sphere. Condone order of subtraction. A1 Magnitude correct.	
c)	M1 Any complete method to find the trig ratio of a relevant angle. A1 $\cos\theta = \frac{4}{5}$, $\tan\frac{\theta}{2} = \frac{1}{3}$, … Or M1 find angle of approach to the line of centres and angle after collision. A1 values correct. (both 71.56 …..) M1 solve for θ A1 37^0 (Q specifies nearest degree) Special case: candidates who act as if the line of centres is in the direction of **i**: CLM u+2v = 8 NIL v-u = 2 u=4/3, v=10/3 4/3i + j ; 10/3i – j Impulse 2m-4/3m = 2/3m $\frac{10+1}{\sqrt{10}\sqrt{\frac{109}{9}}} = \cos\theta$ θ = 1.70^0 Work is equivalent, so treat as a MR: M1A0M1A0M1A1A1 M1A1 M1A1M1A1	

6 (a)	 At E, $\frac{2.45e}{0.5} = 0.1g$ $\Rightarrow e = 0.2$ $\Rightarrow$ $0.5(orl) + 0.2 + y = 2\sin 2t + 0.5(orl) + x$ $\Rightarrow 0.2 + y = 2\sin 2t + x$ *	M1 A1 B1 (3)
(b)	R(↓) $0.1g - T = 0.1\ddot{y}$ $0.1g - \frac{2.45x}{0.5} = 0.1\ddot{y}$ $0.98 - 4.9(0.2 + y - 2\sin 2t) = 0.1\ddot{y}$ $(-4.9y + 9.8\sin 2t = 0.1\ddot{y})$ $\Rightarrow \frac{d^2y}{dt^2} + 49y = 98\sin 2t$ *	M*1 M1 DM*1A1 A1 cso (5)
(c)	CF is $y = A\cos 7t + B\sin 7t$ Hence GS is $y = A\cos 7t + B\sin 7t + \frac{98}{45}\sin 2t$ t = 0, y = o: 0 = A so, $y = B\sin 7t + \frac{98}{45}\sin 2t$ $\dot{y} = 7B\cos 7t + \frac{196}{45}\cos 2t$ t = 0, $\dot{y} = 0 : 0 = 7B + \frac{196}{45}$ $\Rightarrow B = -\frac{28}{45}$ $\Rightarrow y = \frac{14}{45}(7\sin 2t - 2\sin 7t)$	M1 A1 B1 M1 A1 (5)
(d)	$\dot{y} = \frac{14}{45}(14\cos 2t - 14\cos 7t)$ $\dot{y} = 0 \Rightarrow \cos 2t = \cos 7t$ $\Rightarrow 7t = 2k\pi \pm 2t$ k=1 $\Rightarrow$ 9t = 2π (or 5t=2π) $t = \frac{2\pi}{9}$, accept 0.698s, 0.70s.	B1 M1 M1 A1 (4)

a)	M1 Hooke's law to find extension at equilibrium A1 cao B1 Q specifies reference to a diagram. Correct reasoning leading to **given answer**.	
b)	M1 Use of F=ma. Weight, tension and acceleration. Condone sign errors. M1 Substitute for tension in terms of x M1 Use given result to substitute for x in terms of y A1 Correct unsimplified equation A1 Rearrange to **given form** cso.	
c)	M1 Correct form for CF A1 GS for y correct B1 Deduce coefficient of cos θ = 0 M1 Differentiate their y and substitue t=0, $\dot{y} = 0$ A1 y in terms of t. Any exact equivalent.	
d)	B1 $\dot{y}$ correct M1 set $\dot{y} = 0$ M1 solve for general solution for t: $7t = 2k\pi \pm 2t$ or: $\sin\frac{9t}{2} \times \sin\frac{5t}{2} = 0 \Rightarrow \sin\frac{9t}{2} = 0 \, or \sin\frac{5t}{2} = 0$ A1 Select smallest value	

GCE

Edexcel GCE

Mathematics

Mechanics 4 M4 (6680)

June 2008

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Mark Scheme – Final

Edexcel GCE

Mathematics

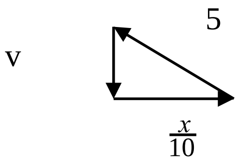
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

June 2008
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.	$_Q\mathbf{V}_P = \mathbf{V}_Q - \mathbf{V}_P = (3\mathbf{i}+7\mathbf{j}) - (5\mathbf{i}-4\mathbf{j})$	M1
	$= (-2\mathbf{i}+11\mathbf{j})$	A1
	$\tan\theta = \frac{11}{2} \Rightarrow \theta = 79.69^0...$	M1 A1
	Bearing is 350^0	A1 **5**
2.	$2m(2\mathbf{i}-2\mathbf{j}) + m(-3\mathbf{i}-\mathbf{j}) = 2m(\mathbf{i}-3\mathbf{j}) + m\mathbf{v}$	M1 A1
	$(\mathbf{i}-5\mathbf{j}) = (2\mathbf{i}-6\mathbf{j}) + \mathbf{v}$	
	$(-\mathbf{i}+\mathbf{j}) = \mathbf{v}$	A1
	$\lvert\mathbf{v}\rvert = \sqrt{(-1)^2+1^2} = \sqrt{2}$ m s^{-1} cwo	DM1 A1 **5**
3.	$mg - mkv = m\frac{dv}{dt}$	M1* A1 A1
	$\int dt = \int \frac{dv}{g-kv}$	DM1*
	$t = -\frac{1}{k}\ln(g-kv) + c$	A1cao
	$t=0, v=u \Rightarrow c = \frac{1}{k}\ln(g-ku)$	M1†
	$T = \frac{1}{k}\ln(g-ku) - \frac{1}{k}\ln(g-2ku)$	DM1†
	$= \frac{1}{k}\ln\left(\frac{g-ku}{g-2ku}\right)$	A1 **8**

Question Number	Scheme	Marks
4.	$u\cos 2\theta = v\cos\theta$	M1 A1
	$\frac{3}{8}u\sin 2\theta = v\sin\theta$	M1 A1
	$3\tan 2\theta = 8\tan\theta$	M1
	$\dfrac{6\tan\theta}{1-\tan^2\theta} = 8\tan\theta$	M1
	$\tan^2\theta = \frac{1}{4} \quad (\tan\theta \neq 0)$	
	$\tan\theta = \frac{1}{2}$	M1 A1 **8**
5.(a)	$-T - \dfrac{1}{2}mg - 2mv\sqrt{\dfrac{g}{l}} = m\ddot{x}$	M1 A3,2,1,0
	$\dfrac{-mgx}{l} - \dfrac{1}{2}mg - 2m\dot{x}\sqrt{\dfrac{g}{l}} = m\ddot{x}$	M1
	$\dfrac{d^2x}{dt^2} + 2\omega\dfrac{dx}{dt} + \omega^2 x = -0.5g \quad$ (AG)	A1 (6)
(b)	$u^2 + 2\omega u + \omega^2 = 0 \Rightarrow u = \omega$ (twice)	
	CF is $x = e^{-\omega t}(At+B)$	B1
	PI is $x = -\dfrac{1}{2}l \quad (-\dfrac{g}{2\omega^2})$	
	GS is $x = e^{-\omega t}(At+B) - \dfrac{1}{2}l$	M1
	$t = 0, x = 0 \Rightarrow B = \dfrac{1}{2}l \quad (\dfrac{g}{2\omega^2})$	M1
	$\dfrac{dx}{dt} = -\omega e^{-\omega t}(At+B) + Ae^{-\omega t}$	M1
	$t = 0, \dfrac{dx}{dt} = \sqrt{gl} = \omega l \Rightarrow A = \dfrac{3}{2}\omega l (= \dfrac{3\sqrt{gl}}{2})(= \sqrt{gl} + \dfrac{0.5g}{\omega})$	M1
	so $x = e^{-\omega t}(\dfrac{3}{2}\omega l t + \dfrac{1}{2}l) - \dfrac{1}{2}l = \dfrac{1}{2}le^{-\omega t}(3\omega t + 1) - \dfrac{1}{2}l$	A1 (6)
(c)	$\dfrac{dx}{dt} = 0 \Rightarrow -\omega e^{-\omega t}(At + B) + Ae^{-\omega t} = 0$	M1
	$\Rightarrow t = \dfrac{2}{3\omega}$	M1 A1 (3)
		15

6.(a)	 vector triangle	M1
	$v^2 + (\frac{x}{10})^2 = 5^2$	M1
	$\Rightarrow 100v^2 = 2500 - x^2$	A1 (3)
(b)	$200v\frac{dv}{dx} = -2x$	M1 A1
	$200\frac{d^2x}{dt^2} + 2x = 0$	D M1
	$\frac{d^2x}{dt^2} + \frac{x}{100} = 0$ *	A1 (4)
(c)	Aux equn: $m^2 + \frac{1}{100} = 0$	M1
	$\Rightarrow m = \pm\frac{i}{10}$	A1
	$x = A\sin\frac{t}{10} + B\cos\frac{t}{10}$	A1
	$t = 0, x = 0 \Rightarrow B = 0$	B1
	$\frac{dx}{dt} = \frac{A}{10}\cos\frac{t}{10}$	M1
	$t = 0, x = 0 \Rightarrow v = \frac{dx}{dt} = 5$	
	$\Rightarrow 5 = \frac{A}{10} \Rightarrow A = 50$	M1
	$\Rightarrow x = 50\sin\frac{t}{10}$	A1
	$x = 30$: $30 = 50\sin\frac{t}{10}$	
	$\Rightarrow t = 10\sin^{-1}(\frac{3}{5}) = 6.44$ s	M1A1 (9)
		16

7.(a)	PE of rod $= -kMga\sin 2\theta$ $BP = 2\text{x}2a\sin\theta = 4a\sin\theta$ PE of mass $= -Mg(6a - 4a\sin\theta)$ $V = -Mg(6a - 4a\sin\theta) - kMga\sin 2\theta$ $= Mga(4\sin\theta - k\sin 2\theta) +$ constant *	B1 M1 A1 M1 A1 (5)
(b)	$\frac{dV}{d\theta} = Mga(4\cos\theta - 2k\cos 2\theta)$ so, $4\text{x}\frac{3}{4} - 2k(2(\frac{3}{4})^2 - 1) = 0$ $\Rightarrow k = 12$	M1 A1 M1 M1 A1 (5)
(c)	$4\cos\theta - 24(2\cos^2\theta - 1) = 0$ $12\cos^2\theta - \cos\theta - 6 = 0$ $(4\cos\theta - 3)(3\cos\theta + 2) = 0$ $\cos\theta = -\frac{2}{3}$	M1 D M1 A1 (3)
(d)	$\frac{d^2V}{d\theta^2} = (Mga)(-4\sin\theta + 4k\sin 2\theta)$ when $\cos\theta = \frac{3}{4}, \frac{d^2V}{d\theta^2} = (Mga) \times 44.97.. \Rightarrow$ stable when $\cos\theta = \frac{-2}{3}, \frac{d^2V}{d\theta^2} = (Mga) \times -50.68.. \Rightarrow$ unstable	M1 A1 M1 A1 A1 (5) **18**

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Mark Scheme (Results)

Summer 2009

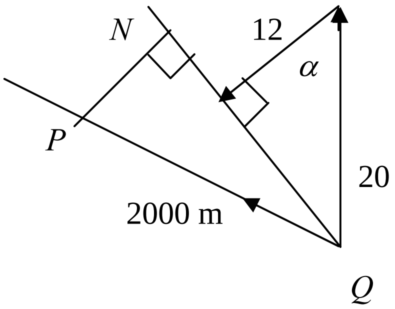
GCE

GCE Mathematics (6680/01)

A PEARSON COMPANY

June 2009
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
Q1	CLM along plane: $v\cos 30^o = u\cos 45^o$	M1 A1
	$v = u\sqrt{\frac{2}{3}}$	A1
	Fraction of KE Lost = $\dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \dfrac{1}{3}$	M1 M1 A1
		[6]
Q2	$-mg - mkv^2 = ma$	M1 A1
	$-(g + kv^2) = v\dfrac{dv}{dx}$	M1
	$\pm\int_0^X dx = \int_{\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}} \dfrac{-v\,dv}{(g+kv^2)}$	DM1 A1 (both previous)
	$X = \dfrac{1}{2k}\left[\ln(g+kv^2)\right]_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\sqrt{\frac{g}{k}}}$	M1 A1
	$= \dfrac{1}{2k}\left(\ln 2g - \ln\dfrac{5g}{4}\right)$	M1
	$= \dfrac{1}{2k}\ln\dfrac{8}{5}$	A1
		[9]

Question Number	Scheme	Marks
Q3 (a)		M1
	$\cos\alpha = \frac{12}{20}$	M1 A1
	Bearing is $180^{\circ} + \alpha = 233^{o}$ (nearest degree)	A1
		(4)
(b)	$PN = 2000\cos(135^{\circ} - \alpha) = 200\sqrt{2}$ m or decimal equivalent	M1A1ft A1
		(3)
(c)	$\sqrt{20^2 - 12^2}$	B1
	Time to closest approach = $\dfrac{QN}{\sqrt{20^2 - 12^2}}$	M1
	$= \dfrac{2000\sin(135^{\circ} - \alpha)}{16}$	A1
	Distance moved by Q = their t x 12	DM1
	$= 1050\sqrt{2}$ m or decimal equivalent	A1
		(5)
		[12]

Question Number	Scheme	Marks
Q4 (a)	$V = -mg2a\sin 2\theta - \frac{7}{20}mg(L - 4a\sin\theta)$	M1 B1 A1
	$= \frac{1}{5}mga(7\sin\theta - 10\sin 2\theta) - \frac{7}{20}mgL$	A1
		(4)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = \frac{1}{5}mga(7\cos\theta - 20\cos 2\theta)$	M1 A1
	$\frac{1}{5}mga(7\cos\theta - 20\cos 2\theta) = 0$	DM1
	$7\cos\theta - 20(2\cos^2\theta - 1) = 0$	DM1
	$40\cos^2\theta - 7\cos\theta - 20 = 0$	A1
	$(5\cos\theta - 4)(8\cos\theta + 5) = 0$	
	$\cos\theta = \frac{4}{5}$ or ($\cos\theta = -\frac{5}{8} \Rightarrow 2\theta > 180^{o}$)	DM1 A1 DM1
		(8)
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{1}{5}mga(-7\sin\theta + 40\sin 2\theta)$	M1 A1
	$= \frac{1}{5}mga(-7\sin\theta + 80\sin\theta\cos\theta)$	
	When $\cos\theta = \frac{4}{5}$,	
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{1}{5}mga(\frac{-21}{5} + 80 \text{x} \frac{3}{5} \text{x} \frac{4}{5}) = \frac{171}{25}mga$	M1
	> 0 therefore stable	A1 cso
		(4)
		[16]

Question Number	Scheme	Marks
Q5 (a)	CLM: $2(\mathbf{i} + 2\mathbf{j}) + -2\mathbf{i} = \mathbf{2\,j} + \mathbf{v}$ $\mathbf{v} = 2\mathbf{j}$ m s^{-1}	M1 A1 A1 (3)
(b)	$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i} + 2\mathbf{j}))$ $= (-2\mathbf{i} - 2\mathbf{j})$ Ns Since **I** acts along l.o.c.c. **,** l.o.c.c is parallel to $\mathbf{i} + \mathbf{j}$	M1 A1 A1 B1 (4)
(c)	Before A: $(\mathbf{i} + 2\mathbf{j}).\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{3}{\sqrt{2}}$ B: $-2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{-2}{\sqrt{2}}$ After A: $\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}$ B: $2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{2}{\sqrt{2}}$	M1 A3
	NIL: $e = \frac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$	DM1 A1 (6) **[13]**

Question Number	Scheme	Marks
Q6 (a)	$(\rightarrow),\ T = m\ddot{y}$	M1
	Hooke's Law:	
	$T = \frac{2mn^2ax}{2a} = mn^2x$	B1
	$\left.\begin{array}{l} x + y = \frac{1}{2}ft^2 \\ \dot{x} + \dot{y} = ft \\ \ddot{x} + \ddot{y} = f \end{array}\right\}$	B2
	$so,\quad (\rightarrow),\ mn^2x = m\ddot{y} = m(f - \ddot{x})$	DM1
	$\ddot{x} + n^2x = f$ **	A1
		(6)
(b)	C.F. : $x = A\cos nt + B\sin nt$	B1
	P.I. : $x = \frac{f}{n^2}$	B1
	Gen solution: $x = A\cos nt + B\sin nt + \frac{f}{n^2}$	M1
	$\dot{x} = -An\sin nt + Bn\cos nt$ follow their PI	M1 A1ft
	$\left.\begin{array}{l} t = 0, x = 0 \Rightarrow A = -\frac{f}{n^2} \\ t = 0, \dot{x} = 0 \Rightarrow B = 0 \end{array}\right\}$	M1 A1
	$x = \frac{f}{n^2}(1 - \cos nt)$	A1
		(8)
(c)	$\dot{x} = 0 \Rightarrow \quad nt = \pi$	M1
	$x_{\max} = \frac{f}{n^2}(1 - -1) = \frac{2f}{n^2}$	M1 A1
		(3)
(d)	$\dot{y} = ft - \dot{x}$	M1
	$= f\frac{\pi}{n} - 0 = \frac{f\pi}{n}$	A1
		(2)
		[19]

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Mark Scheme (Results)

Summer 2010

GCE

GCE Mechanics M4 (6680/01)

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Summer 2010

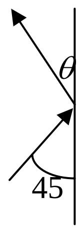
Publications Code UA024478

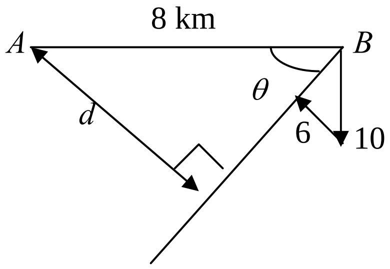
Mechanic guidance:

- For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
- Omission of *g* from a resolution is an accuracy error, not a method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Where there is only one method mark for a question or part of a question, this is for a *complete* method.
- Omission of units is not (usually) counted as an error.
- Use of 9.81 for g is a rubric error. Deduct the final A1 from the first part of any question affected.
- More than 3 sf in an answer using an approximation for g is an accuracy error. Deduct the final A1 from the first part of any question affected.
- A dimensionally incorrect equation is a method error unless a correct equation was quoted and the error arises from a slip in substitution of values.
- For a misread which does not alter the character of a question or materially simplify it, all marks in that part of the question affected become ft. Deduct the first 2 A or B marks gained as a result and give the rest.

Summer 2010
Mechanics M4 6680
Mark Scheme

Question Number	Scheme	Marks
Q1	$v(3\mathbf{i}-4\mathbf{j})=\mathbf{v}_W-u\mathbf{j}$	M1A1
	$\mathbf{v}_W=3v\mathbf{i}+(u-4v)\mathbf{j}$	
	$w\mathbf{i}=\mathbf{v}_W-\frac{u}{5}(-3\mathbf{i}+4\mathbf{j})$	M1A1
	$\mathbf{v}_W=(w-\frac{3u}{5})\mathbf{i}+\frac{4u}{5}\mathbf{j}$	
	$(u-4v)=\frac{4u}{5}$	M1
	$v=\frac{u}{20}$	A1
	$\mathbf{v}_W=\frac{3u}{20}\mathbf{i}+\frac{4u}{5}\mathbf{j}$	A1
		7

Question Number	Scheme	Marks
Q2 (a)	$\uparrow 2$ $\uparrow 1$ $1 \leftarrow$ $\rightarrow 1$ S 0.3kg T 0.6 kg $2 \uparrow$ $\uparrow 1$ $\rightarrow v$ $w \leftarrow$	
	$0.3v - 0.6w = 0.3$	M1 A1
	$v - 2w = 1$	
	$\frac{1}{2}\ (v + w) = 2$	M1 A1
	$v + w = 4$	
	$w = 1, v = 3$	
	(i) $\mathbf{u_1} = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$	A1 A1
		(6)
(b)	$\uparrow 1$ $v \leftarrow$	
	$v = 0.5$	B1
	$1 \uparrow$ $\rightarrow 1$	
	 	
	$\tan\theta = 0.5$ tan θ = their v	M1
	$\theta = 26.6$	A1
	their θ + 45º	M1
	Defln angle $= 45 + 26.6 = 71.6^{\circ}$	A1
		(5)
(c)	KE Loss = $\frac{1}{2}$ x 0.6 x $\left\{(1^2 + 1^2) - (1^2 + v^2)\right\}$	M1 A1
	$= 0.225$ J	A1
		(3) **14**

Question Number	Scheme	Marks
Q3 (a)		M1
	$\cos\theta = \frac{6}{10} \Rightarrow \theta = 53.1^{\circ}$	M1 A1
	Bearing is 307°	A1
		(4)
(b)	$d = 8 \sin\theta$ (=8×0.8)	M1 A1
	= 6.4 km	A1
		(3)
(c)	$T = \frac{8\cos\theta}{\sqrt{10^2 - 6^2}}$	M1 A1
	$= 0.6$ hrs	
	i.e. the time is 12:36 pm	A1
		(3)
		10

Question Number	Scheme	Marks
Q4		
(a)	$-mg(1+\frac{v^2}{k^2}) = m\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1
	$g\int_0^T \mathrm{d}t = \int_U^0 \frac{-k^2\mathrm{d}v}{(k^2+v^2)}$	DM1
	$T = \frac{k}{g}\left[\tan^{-1}\frac{v}{k})\right]_0^U$	A1
	$= \frac{k}{g}\tan^{-1}\frac{U}{k}$	DM1A1
		(6)
(b)	$-mg(1+\frac{v^2}{k^2}) = mv\frac{\mathrm{d}v}{\mathrm{d}x}$	M1 A1
	$g\int_0^H \mathrm{d}x = \int_U^0 \frac{-k^2v\mathrm{d}v}{(k^2+v^2)}$	DM1
	$H = \frac{k^2}{2g}\left[\ln(k^2+v^2\right]_0^U$	A1
	$H = \frac{k^2}{2g}\ln\frac{(k^2+U^2)}{k^2}$	DM1A1
		(6)
		12

Question Number	Scheme	Marks
Q5		
(a)	$\sqrt{4a^2+16a^2-16a^2\sin\theta}$	M1 A1
	Let length of string be L.	
	$V=-4mga\cos\theta-mg(L-\sqrt{4a^2+16a^2-16a^2\sin\theta}$	M1 A1
	$=-4mga\cos\theta-mgL+2mga\sqrt{5-4\sin\theta}$	
	$=2mga\left\{\sqrt{5-4\sin\theta}-2\cos\theta\right\}+\text{constant}$ **	A1
		(5)
(b)	$V'(\theta)=2mga\left\{\dfrac{-2\cos\theta}{\sqrt{5-4\sin\theta}}+2\sin\theta\right\}$	M1 A1
	For equilibrium, $V'(\theta)=0$	
	$\left\{\dfrac{-2\cos\theta}{\sqrt{5-4\sin\theta}}+2\sin\theta\right\}=0$	M1
	$\dfrac{\cos^2\theta}{5-4\sin\theta}=\sin^2\theta$	
	$1-\sin^2\theta=\sin^2\theta(5-4\sin\theta)$	DM1
	$4\sin^3\theta-6\sin^2\theta+1=0$ **	A1
		(5)
(c)	$V''(\theta)=2mga(\dfrac{\left\{\sqrt{5-4\sin\theta}.2\sin\theta-\dfrac{-2\cos\theta.(-4\cos\theta)}{2\sqrt{5-4\sin\theta}}\right\}}{(5-4\sin\theta)}+2\cos\theta)$	M1 A1 A1
	$V''(\dfrac{\pi}{6})=2mga\left\{\dfrac{\sqrt{3}-\dfrac{8\text{x}\frac{3}{4}}{2\sqrt{3}}}{3}+\sqrt{3}\right\}=2mga\sqrt{3}>0$ so stable	DM1 A1
		(5)
		15

Question Number	Scheme	Marks
Q6		
(a)	$T_1 = \frac{2mge}{a}$; $T_2 = \frac{mg(2a-e)}{a}$	B1 (either)
	$T_1 = T_2$	
	$2e = (2a-e)$	M1 A1
	$e = \frac{2a}{3}$	
	$AP = a + \frac{2a}{3} = \frac{5a}{3}$ **	A1
		(4)
(b)	$T_2 - T_1 - 4m\omega\dot{x} = m\ddot{x}$	
	$\frac{mg}{a}\left(\frac{4a}{3} - x\right) - \frac{2mg}{a}\left(\frac{2a}{3} + x\right) - 4m\omega\dot{x} = m\ddot{x}$	M1 A3
	$\ddot{x} + 4\omega\dot{x} + \frac{3g}{a}x = 0$	
	$\ddot{x} + 4\omega\dot{x} + 3\omega^2 x = 0$ **	A1
		(5)
(c)	$\lambda^2 + 4\omega\lambda + 3\omega^2 = 0$	
	$(\lambda + 3\omega)(\lambda + \omega) = 0$	M1
	$\lambda = -3\omega$ or $\lambda = -\omega$	
	$x = Ae^{-\omega t} + Be^{-3\omega t}$	A1
	$\dot{x} = -\omega Ae^{-\omega t} - 3\omega Be^{-3\omega t}$	M1 A1
	$t = 0$, $x = \frac{1}{2}a$, $\dot{x} = 0$	M1
	$\frac{1}{2}a = A + B$	
	$0 = -\omega A - 3\omega B$	A1
	$A = \frac{3}{4}a$, $B = -\frac{1}{4}a$	A1
	$\dot{x} = v = \frac{3}{4}a\omega\,(\mathrm{e}^{-3\omega t} - \mathrm{e}^{-\omega t})$	A1
		(8)
		17

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Mark Scheme (Results)

June 2011

GCE Mechanics M4 (6680) Paper 1

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June 2011
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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

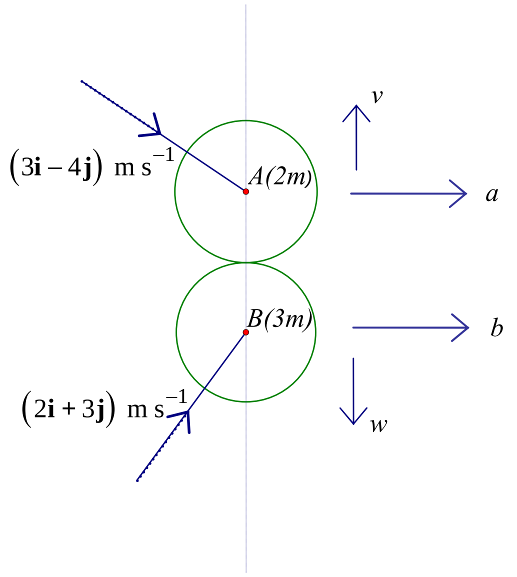
   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

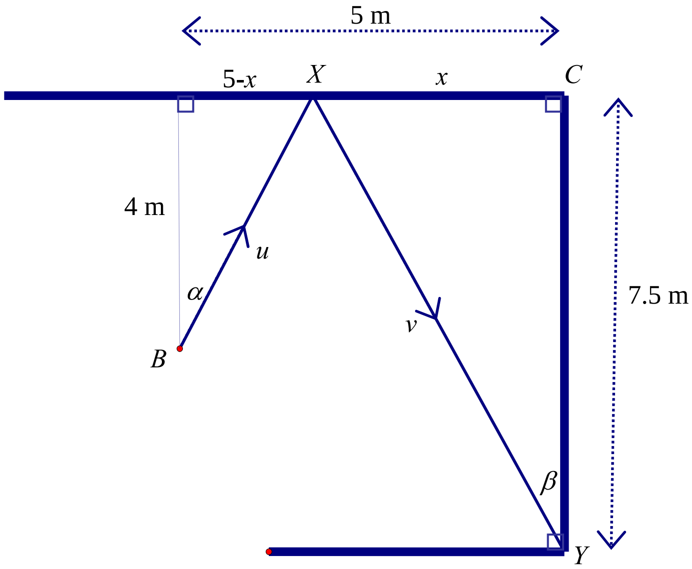
3. Abbreviations

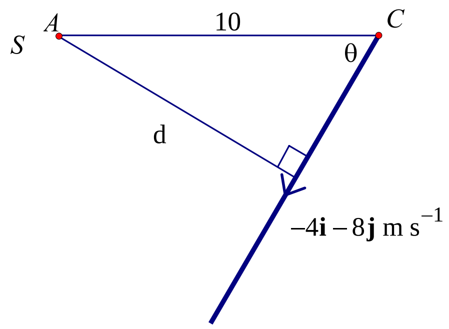
   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

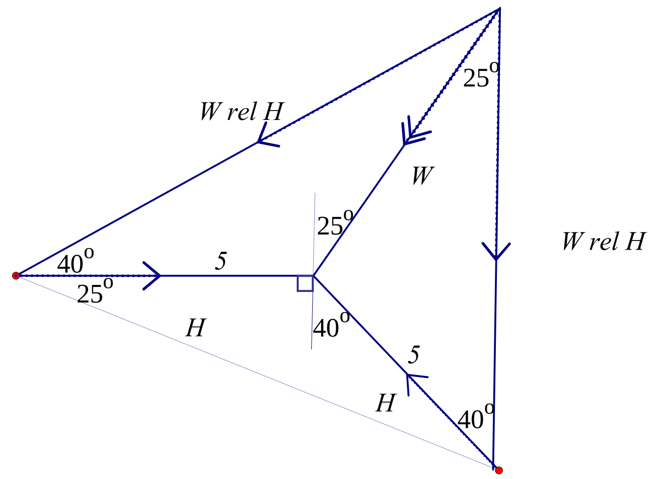
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - □ The second mark is dependent on gaining the first mark

June 2011
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.		
	$\leftrightarrow a = 3$ & $b = 2$	B1
	b Conservation of linear momentum: $-4\times 2 + 3\times 3 = 2v - 3w (=1)$	M1A1
	Restitution: $v + w = \mathrm{e}\times 7 \ (=3)$	M1A1
	Solve the simultaneous equations	DM1
	giving $v = 2$ and $w = 1$	A1
	KE lost $= \frac{1}{2}\times 2m\times((16+9)-(4-9)) + \frac{1}{2}\times 3m\times((9+4)-(1-4))$	M1A1
	$= 24m \ (\mathrm{J})$	A1
		10

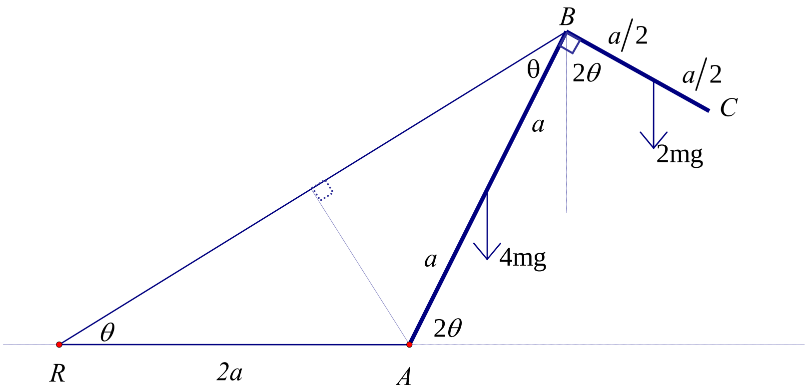
Question Number	Scheme	Marks
2.		
	At X: $\leftrightarrow$ $u\sin\alpha = v\sin\beta$	M1A1
	$\updownarrow$ $v\cos\beta = eu\cos\alpha$	M1A1
	$4v\cos\beta = 3u\cos\alpha$	
	Eliminate u & v by dividing: $\frac{\tan\alpha}{3} = \frac{\tan\beta}{4}$	M1
	Substitute for the trig ratios: $\frac{5-x}{3\times 4} = \frac{x}{4\times 7.5}$	DM1A1
	Solve for x: $37.5 - 7.5x = 3x$	DM1
	$x = 3.57$ (m) or better, $\frac{25}{7}$	A1
		9
3. (a)	Velocity of C relative to S $= (8\mathbf{i} + u\mathbf{j}) - (12\mathbf{i} + 16\mathbf{j})$	M1
	$= (-4\mathbf{i} + (u-16)\mathbf{j})\ (\text{m s}^{-1})$	A1
		(2)
(b) (i)	C intercepts S $\Rightarrow$ relative velocity is parallel to **i**.	
	$\Rightarrow u - 16 = 0,\ u = 16$	M1A1
		(2)
(ii)	10 km at 4 km h^{-1} takes 2.5 hours, so 2.30pm	M1A1
		(2)

Question Number	Scheme	Marks
(c)	$u=8$, relative velocity $=-4\mathbf{i}-8\mathbf{j}$.	B1
		
	Correct distance identified	B1
	Using velocity: $\tan\theta=\frac{8}{4}=2\Rightarrow\sin\theta=\frac{2}{\sqrt{5}}$	
	Using distance: $\sin\theta=\frac{d}{10}=\frac{2}{\sqrt{5}}$,	M1A1
	$\text{d}=\frac{20}{\sqrt{5}}=4\sqrt{5}=8.9\ (\text{km})$	A1 (5)
		11

Question Number	Scheme	Marks
4. **(a)**	 2 vector triangles with a common side …..correct and drawn on a single diagram Wind is from bearing 025°, (N 25° E)	M1 A1 A1 (3)
(b)	$\frac{5}{\sin 25^{\circ}} = \frac{W}{\sin 40^{\circ}}$ (ft on their 25°) $W = \frac{5 \times \sin 40^{\circ}}{\sin 25^{\circ}} = 7.6 \left(\text{km h}^{-1}\right)$	M1A1ft M1A1 (4)

Question Number	Scheme	Marks
5.		
(a)	Need an equation linking speed and displacement, so	
	$mv\frac{dv}{dx} = -(a+bv^2)$	M1A1
	Separating the variables: $\int \frac{6v}{a+bv^2}dv = \int -1dx$	M1
	Integrating : $\frac{3}{b}\ln(a+bv^2) = -x+(C)$	A1
	$X = \frac{3}{b}\left[\ln(a+bU^2) - \ln(a)\right] = \frac{3}{b}\ln\left[1+\frac{bU^2}{a}\right]$ **	M1A1
	as required	(6)
(b)	Equation connecting v and t: $6\frac{dv}{dt} = -(12+3v^2)$	M1
	Separate the variables: $\int \frac{-6}{12+3v^2}dv = \int 1dt$	M1, A1
	$\int_U^0 \frac{-2}{4+v^2}\mathrm{d}v = \int_0^U \frac{2}{4+v^2}\mathrm{d}v = T$	M1
	$T = \frac{2}{2}\tan^{-1}\frac{U}{2} = \tan^{-1}\frac{U}{2}(\mathrm{s})$	A1
		(5)
		11

Question Number	Scheme	Marks
6. **(a)**	Using F = ma: $4\dfrac{d^2x}{dt^2} = -9x - 12v$	M1A1
	$= -9x - 12\dfrac{dx}{dt}$	M1
	Hence $4\dfrac{d^2x}{dt^2} + 12\dfrac{dx}{dt} + 9x = 0$ **	A1
		(4)
(b)	Auxiliary eqn : $4m^2 + 12m + 9 = 0$,	B1
	$(2m+3)^2 = 0, m = -3/2,\ \lambda = 3/2$	B1
	$t = 0, x = 4 \Rightarrow B = 4$	B1
	$t = 0,\ \dot{x} = e^{-\lambda t}\left(-\lambda\left(At + B\right) + A\right) = 0 \Rightarrow -6 + A = 0,\ A = 6$	B1
		(4)
(c)	$\dot{x} = e^{-\frac{3}{2}t}(-\frac{3}{2}(6t+4)+6) = -9te^{-\frac{3}{2}t}$	M1A1
	$\ddot{x} = e^{-\frac{3}{2}t}(-9-(-9t)\times\frac{3}{2})$,	M1
	so acceleration = 0 when $t = 2/3$	
	at which time, $v = -6e^{-1}$, so max speed $= 6/e \approx 2.21$ m s^{-1} (3sf)	A1, A1
		(5) **13**

Question Number	Scheme	Marks
7. **(a)**		
	$\text{BR} = 2\times 2a\cos\theta = 4a\cos\theta$	B1
	$\text{EPE} = 3mg\dfrac{(4a\cos\theta)^2}{2\times 2a}$	M1
	$=12mga\cos^2\theta = 6mga + 6mga\cos 2\theta$	A1
	GPE: taking AR as the level of zero GPE,	
	GPE = GPE of AB + GPE of BC	M1+M1
	$= 4mg\times a\sin 2\theta + 2mg\left(2a\sin 2\theta - a/2\cos 2\theta\right)$	A1
	$= 8mga\sin 2\theta - mga\cos 2\theta$	
	$\Rightarrow$ Total $V = 8mga\sin 2\theta + 5mga\cos 2\theta + \text{constant},$ as required. **	A1
		(7)
(b)	$\dfrac{dV}{d\theta} = 16mga\cos 2\theta - 10mga\sin 2\theta$	M1 A1
	$\dfrac{dV}{d\theta} = 0 \Rightarrow 10\sin 2\theta = 16\cos 2\theta$	M1
	$\Rightarrow \tan 2\theta = \dfrac{8}{5} \Rightarrow \theta = 0.51$ radians (29.0°)	A1
		(4)
	Or: $8mga\sin 2\theta + 5mga\cos 2\theta = \sqrt{89}mga\cos(2\theta-\alpha),\ \tan\alpha = \dfrac{8}{5}$	M1A1
	t. pts when $2\theta - \alpha = n\pi \Rightarrow \theta = 0.51$ rads.	M1A1
(c)	$\dfrac{d^2V}{d\theta^2} = -32mga\sin 2\theta - 20mga\cos 2\theta$	M1
	$\theta = 0.51 \Rightarrow \dfrac{\mathrm{d}^2V}{\mathrm{d}\theta^2} < 0$, equilibrium is unstable. cso	M1A1
		(3) **14**
	Or: $2\theta - \alpha = 0 \Rightarrow \cos(2\theta - \alpha) = 1$	
	Max value $\Rightarrow$ equilibrium is unstable	

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GCE Mechanics M4
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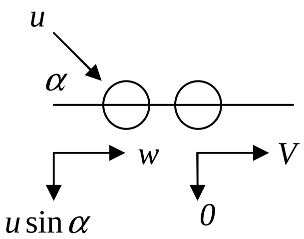
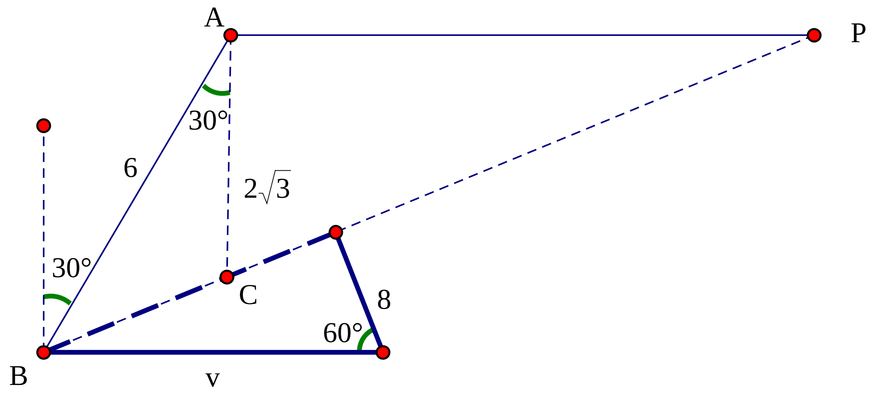
Summer 2012
Publications Code UA032684

June 2012
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks	Notes
1. (a)			
	$mu\cos\alpha = mw + 2mV$	M1 A1	CLM parallel to the line of centres. $\frac{4}{5}u = w + 2V$. Need all terms but condone sign errors.
	$eu\cos\alpha = -w \;+\; V$	M1 A1	Impact law. Must be the right way round. $\frac{3}{4}\times\frac{4}{5}u = V - w$
	$u\cos\alpha(e+1) = 3V \Rightarrow$ (i) $u = \frac{15V}{7}$	M1 A1	Eliminate *w* and solve for *u* in terms of *V* or v.v. 2.14*V* or better
	$\Rightarrow w = -\frac{2V}{7}$	A1	Solve for *w* in terms of *V*. -0.286*V* or better
	(ii) speed of *S* $= \sqrt{(\frac{-2V}{7})^2 + (u\sin\alpha)^2} = \frac{V\sqrt{85}}{7}$	M1 A1 (9)	Use of Pythagoras with their $u\sin\alpha$ and *w*. $\sqrt{\left(\frac{-2V}{7}\right)^2 + \left(\frac{15V}{7}\times\frac{3}{5}\right)^2}$ $\sqrt{\frac{85}{49}}V$, accept 1.32*V* or better

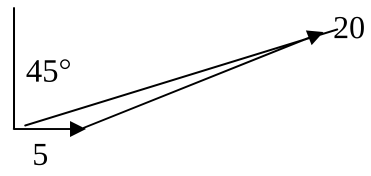
Question Number	Scheme	Marks	Notes
(b)	$\tan\theta = \dfrac{\frac{9V}{7}}{\frac{2V}{7}} = \frac{9}{2}$	M1	Direction of *S* after the collision. Condone $\dfrac{2}{9}$
		A1	$77.5°$ or $12.5°$ seen or implied
	defln angle = $180° - (\theta + \alpha)$	DM1	Combine their θ and α to find the required angle. e.g. $12.5° + \tan^{-1}\left(\dfrac{4}{3}\right)$
	$= 65.7°$ (3 sf)	A1	Accept 66°
		(4)	
		13	

Question Number	Scheme	Marks	Notes
2.	With B as origin,		
	$\mathbf{r}_A = (6\sin 30\mathbf{i} + 6\cos 30\mathbf{j})$	M1	Express the original relative positions in component (vector) form – one term correct.
	$= (3)\mathbf{i} + (3\sqrt{3})\mathbf{j}$	A1	Both terms correct (substitution of trig values not required).
	$\mathbf{r}_B = vt\mathbf{i}$ or $\mathbf{v}_B = v\mathbf{i}$	B1	Position of B at time t (seen or implied)
	$(v-4)\mathbf{i} + (4\sqrt{3})\mathbf{j}$	M1	Express the relative velocity in component form – one term correct.
	or $\left(v - 8\sin 30\right)\mathrm{i} + \left(8\cos 30\right)\mathrm{j}$	A1	Both terms correct
	When B is $2\sqrt{3}$ km south of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = -2\sqrt{3} \Rightarrow t = \frac{1}{4}$	M1 A1	Compare **j** displacement with $\pm 2\sqrt{3}$ and solve for t cao
	$vt - 3 - 4t = 0 \quad \Rightarrow v = 16$	M1 A1	Equate **i** displacement to zero and substitute their value of t. cao
	When B is due east of A,		
	$-3\sqrt{3} + 4\sqrt{3}t = 0 \Rightarrow t = \frac{3}{4}$ i.e. at 12.45 pm	M1 A1	Equate **j** displacement to zero and solve for t. Any equivalent form for the time.
	then distance AB = $16\text{x}\frac{3}{4} - 3 - 4\text{x}\frac{3}{4} = 6$ km.	M1 A1	Substitute their v & t in the **i** displacement and evaluate cao. Must be a scalar.
		13	
			See over page for geometrical alternative

or	Triangle *ABC*: cosine rule gives $BC^2 = 36 + 12 - 2 \times 6 \times 2\sqrt{3}\cos 30$	M1A1	The given information provides us with two triangles - velocities in bold. Fix *A* and *B* follows the path *BP*. *C* is the point when *B* is due South of *A*, and *P* when it is due East. 
	Solve for *BC* and $\angle ABC$ $BC = 2\sqrt{3}$, $\rightarrow$ triangle is isosceles	M1A1	
	$\angle B$ in velocity triangle is 30°	B1	
	Trig in rt∠ triangle gives relative velocity $=8 \times \tan 60 = 8\sqrt{3}$	M1A1	
	∠APB = 30° (angles of a triangle) so triangle is isosceles and distance *AP* = 6km	M1A1	
	Using cosine rule or symmetry of isosceles triangle, distance *BP* = $6\sqrt{3}$	M1A1	
	Time taken = $\frac{6\sqrt{3}}{8\sqrt{3}} = \frac{3}{4}$ hr, time is now 12.45	M1A1	

3. (a)	$2mg - T - kv^2 = 2ma$	M1 A1	Equation of motion for particle of mass $2m$ aef
	$T - mg - kv^2 = ma$	M1 A1	Equation of motion for particle of mass m aef
	Adding, $mg - 2kv^2 = 3ma$		
	$\frac{2g}{3} - \frac{4kv^2}{3m} = 2v\frac{dv}{dx}$	DM1	Eliminate T, substitute for a and rearrange. Dependent on both previous M marks.
	$\frac{d(v^2)}{dx} + \frac{4kv^2}{3m} = \frac{2g}{3}$ *	A1	Reach **given answer** correctly
		(6)	
(b)	$IF = e^{\int \frac{4k}{3m}dx} = e^{\frac{4kx}{3m}}$	B1	
	$v^2 e^{\frac{4kx}{3m}} = \frac{2g}{3}\int e^{\frac{4kx}{3m}}dx = \frac{mg}{2k}e^{\frac{4kx}{3m}} \left(+C\right)$	M1 A1	Use integrating factor to obtain $\frac{d}{dx}\left(v^2 e^{\frac{4kx}{3m}}\right) = \frac{2g}{3}e^{\frac{4kx}{3m}}$ and integrate
	$v^2 = \frac{mg}{2k} + Ce^{\frac{-4kx}{3m}}$		
	$x = 0, v = 0 \Rightarrow C = -\frac{mg}{2k}$	M1	Use initial values to evaluate C or as limits in a definite integral and find an expression for v^2.
	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$	A1	aef.
		(5)	
OR	Separate variables: $\int \frac{3m}{2mg - 4kv^2}dv^2 = \int 1dx$	B1	CF $v^2 = Ae^{-\frac{4k}{3m}x}$
	$x = -\frac{3m}{4k}\ln\left\|2mg - 4kv^2\right\|\left(+C\right)$	M1A1	PI $v^2 = b \Rightarrow 0 + \frac{4k}{3m}b = \frac{2g}{3}$; GS $v^2 = Ae^{-\frac{4k}{3m}x} + \frac{mg}{2k}$
	$x = -\frac{3m}{4k}\ln\left\|\frac{2mg}{2mg - 4kv^2}\right\|$	M1	$x = 0,\ v = 0 \Rightarrow A = -\frac{mg}{2k}$
	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$	A1	$v^2 = \frac{mg}{2k}(1 - e^{\frac{-4kx}{3m}})$

(c)	When $x = 0, T = \dfrac{4mg}{3}$	M1 A1	Substitute v = 0 in the initial equations and solve for T
	As $x \to \infty, T \to \dfrac{9mg}{6} = \dfrac{3mg}{2}$	M1 A1	For large x, $v^2 \to \dfrac{mg}{2k}$. Substitute in the initial equations and solve for T
	Hence, $\dfrac{4mg}{3} \le T < \dfrac{3mg}{2}$. *	A1	cwo – **answer is given**.
		(5)**16**	

4.(a)			
	$\frac{\sin\theta}{5}=\frac{\sin 45}{20}$	M1	Use a vector triangle to find θ. Condone the 5 ms^{-1} in the wrong direction.
		A1	Correct equation for θ
	$\theta = 10.182...$ Bearing is $45^\circ - \theta = 34.8 = 35^\circ$ (nearest degree)	M1	Use their angle correctly in their triangle to find the bearing.
		A1 (4)	Accept alternative forms e.g. N 35 E
OR	$\text{SW} \rightarrow (20\sin\theta)T = (5+20\cos\theta)T$	M1	45° rt angle triangle
	$3t^2+8t-5=0$, $t=\frac{-8+\sqrt{124}}{6}=0.5225...$	A1	t substitution leading to correct equation in t, use of $R\cos(\theta+\alpha)$ o.e.
	$\theta = 55.18...$ Bearing is $90-\theta = 34.8^\circ$	M1A1 (4)	
(b)	$v^2 = 5^2+20^2-2\text{x}5\text{x}20\cos 124.818...$ OR $v=\frac{20}{\sin 45}\times\sin 124.8$ OR $v = 5\cos 45 + 20\cos\theta$	M1	Complete method to find v
	$v = 23.22$	A1	Or better $\left(\frac{5\sqrt{2}+5\sqrt{62}}{2}\right)$
	$t=\frac{15}{23.22}$ = 0.646 h= 39 min (nearest min)	M1	$\frac{15}{\text{their } v}$
		A1 (4)	The Q specifies "nearest minute"
(c)	Due N, (since current affects both equally)	B1 (1)	cao
(d)	$t=\frac{4}{20}$ = 0.2 h = 12 min	B1 (1)**10**	cso

5. **(a)**		B1		GPE of rod e.g. $-Wa\cos 2\theta$
	$V = -Wa\cos 2\theta + \frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$	M1		GPE of the particle e.g. $\frac{1}{2}W\{3a-(L-6a\cos\theta-4a)\}$ Condone 3a term missing.
		A1		Correct expression including the 3a (unless in the GPE for the rod) Accept aef e.g. $\sqrt{18a^2(1+\cos 2\theta)}$ for $6a\cos\theta$
	$= -Wa\cos 2\theta + 3Wa\cos\theta + (\frac{7Wa}{2} - \frac{WL}{2})$			
	$= Wa(3\cos\theta - \cos 2\theta) + \text{constant}$ *	A1		Obtain the **given answer** correctly
			(4)	
(b)	$\frac{dV}{d\theta} = Wa(-3\sin\theta + 2\sin 2\theta)$	M1		Differentiate the given V wrt θ
		A1		correct
	For equilibrium, $Wa(-3\sin\theta + 2\sin 2\theta) = 0$			
	$\sin\theta(4\cos\theta - 3) = 0$	DM1		Set their derivative = 0
		A1		First answer
	$\Rightarrow \theta = 0$ or $\theta = \cos^{-1}\left(\frac{3}{4}\right)$	A1		Second answer - ignore $\theta = -\cos^{-1}\left(\frac{3}{4}\right)$. 0.72 rads or better
	$\frac{d^2V}{d\theta^2} = Wa(-3\cos\theta + 4\cos 2\theta)$	M1		Obtain the second derivative of V and substitute one of their values for θ
	$\theta = 0 : \frac{d^2V}{d\theta^2} = Wa > 0 \Rightarrow$ *stable*	A1		Correct working and conclusion for one value
	$\theta = \cos^{-1}\frac{3}{4} : \frac{d^2V}{d\theta^2} = -\frac{7Wa}{4} < 0 \Rightarrow$ *unstable*	A1		Correct working and reasoning for the second. ISW for work on $-\cos^{-1}\left(\frac{3}{4}\right)$
			(8)	
			12	

Question	Scheme	Marks		Notes
6.(a)	$T_1 = mg + T_2$	M1		No resultant force and use of Hooke's law
	$\frac{3mge}{a} = mg + \frac{mg(2a-e)}{a}$	A1		Correct equation in one unknown $\frac{3mg(AP-a)}{a} = mg + \frac{mg(3a-AP)}{a}$, $3AP - 3a = a + 3a - AP$
	$e = \frac{3a}{4} \Rightarrow AP = \frac{7a}{4}$ *	A1		Derive **given result** correctly.
				Condone verification for 3/3
			(3)	
(b)	$mg + T_2 - T_1 - mkv = m\ddot{x}$	M1		Equation of motion – requires all terms but condone sign errors.
		A1		o.e. Correct equation in T_1 & T_2.
	$mg + \frac{mg(\frac{5}{4}a - x)}{a} - \frac{3mg(\frac{3}{4}a + x)}{a} - mkv = m\ddot{x}$	DM1		Use Hooke's law with extensions of the form $ka \pm x$
		A1		o.e. Correct unsimplified
	$\ddot{x} + k\dot{x} + \frac{4g}{a}x = 0$ *	A1		**Given answer** derived correctly
			(5)	
(c)	For a damped oscillation, $k^2 < \frac{16g}{a}$	M1		AE will have complex roots
		A1		Correctly substituted inequality
	i.e. $k < 4\sqrt{\frac{g}{a}}$	A1		Only (Q gives k>0) $-4\sqrt{\frac{g}{a}} < k < 4\sqrt{\frac{g}{a}}$ is A0.
			(3)	
			11	

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Rewarding Learning

Mark Scheme (Results)

Summer 2013

GCE Mechanics 4 (6680/01R)

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Summer 2013
Publications Code UA036436

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Rules for Marking Mechanics

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is accuracy error not method error.

- Omission of mass from a resolution is method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

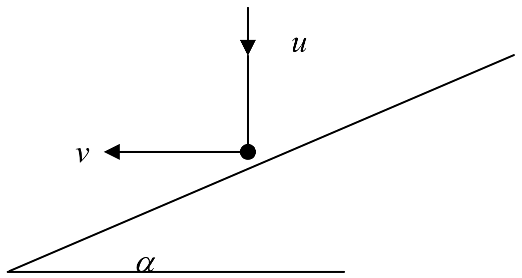
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.

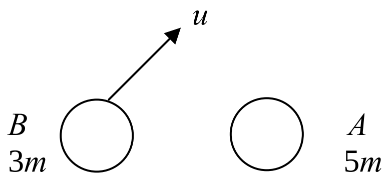
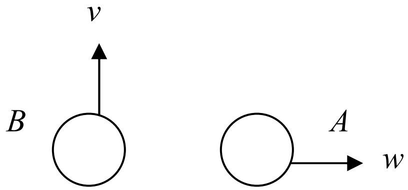
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

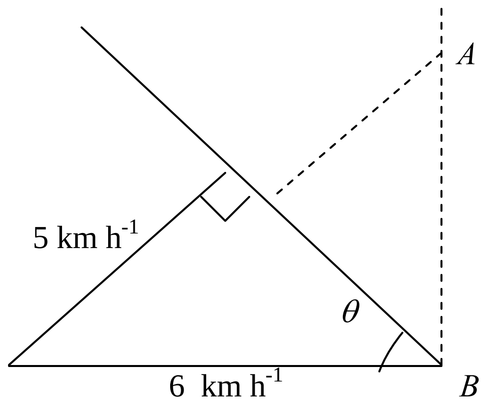
- Accept column vectors in all cases.

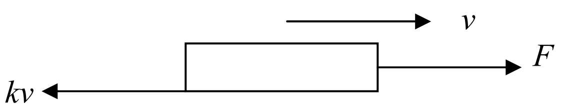
Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

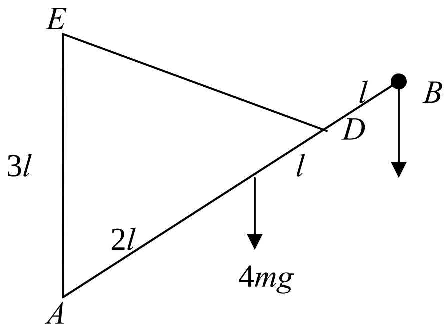
Question Number	Scheme	Marks	
1. (a)	${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$		
	$= -3\mathbf{i} + 9\mathbf{j}$ km h^{-1}	M1	
	Mag $= \sqrt{9+81} = 3\sqrt{10}$	M1A1	9.5 or better
		(3)	
(b)	$\tan\theta = \frac{3}{9}$	M1	Allow ± or reciprocal
	$\theta = 18.4°$		Or 71.6°
	Direction = $360 - 18.4$		
	$= 342°$	A1	Allow 341.6°
		(2)	
		[5]	

Question Number	Scheme	Marks	
2.			
	CLM: $u\sin\alpha = v\cos\alpha$	M1 A1	Must be in correct direction but condone trig confusion
	Impact: $\frac{1}{3}u\cos\alpha = v\sin\alpha$	M1 A1	Condone consistent trig confusion
	$\frac{1}{3}\times\frac{1}{\tan\alpha} = \tan\alpha$	M1	
	$\tan\alpha = \frac{1}{\sqrt{3}}$		
	$\alpha = 30°$ $\left(\text{or } \frac{\pi}{6} \text{ or } 0.52\,\text{rad}\right)$	A1	
		(6) **[6]**	

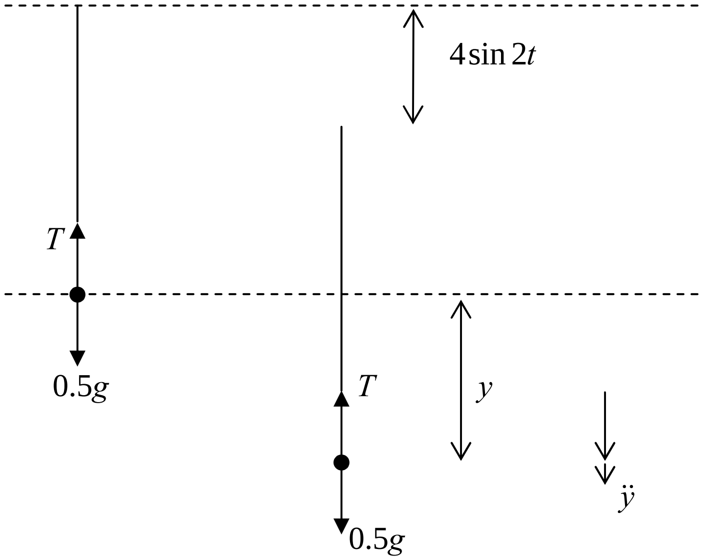
Question Number	Scheme	Marks	
3.			
(a)			
	After impact *B* moves perpendicular to the line of centres	B1	can be implied by appropriate use of θ in an equation, or seen on the diagram
	Perp. to line of centres: $v = u\sin 60 = u\frac{\sqrt{3}}{2}$	M1A1	
		(3)	
(b)	Parallel to line of centres:		
	Con of Mom $3mu\cos 60 + 5m\times 0 = 3m\times 0 + 5mw$	M1A1	
	N.L.R. $eu\cos 60 = w$	M1A1	
	$\frac{1}{2}eu = w$ & $\frac{3}{2}u = 5w$		
	$\rightarrow \frac{1}{2}eu = \frac{3}{10}u$	DM1	Dependent on the two previous M marks
	$e = \frac{3}{5}$	A1	
		(6) **[9]**	

Question Number	Scheme	Marks	
4.		B1	Right angled triangle with the right angle opposite the 6 seen in diagram or implied in working
(a)	$\sin\theta = \frac{5}{6}$	M1	Correct trig.
	$\theta = 56.44...$	A1	
	Bearing = 056°	A1	Allow 56.4°
		(4)	
(b)	Least distance = $4\cos\theta = \frac{(4\sqrt{11})}{6}$ or 2.211 km oe	M1 A1	Correct for their angle 2.2 or better
		(2)	
(c)	${}_Bv_A = \sqrt{6^2 - 5^2} = \sqrt{11}$	B1	3.32
	$t = \frac{4\sin\theta}{\sqrt{11}} \quad (=1.0050...)$	M1 A1ft	Condone consistent trig confusion Ft on their √11
	time = 11 am	B1	
		(4) **[10]**	

Question Number	Scheme	Marks	
5.			
(a)	$Fv = 40000$		
	$1200\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{40000}{v} - kv$	M1 A1	
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 0.3 \qquad 1200 \times 0.3 = \frac{40000}{40} - 40k$	M1	Use initial conditions to find k
	$k = 16$	A1	
	$1200\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{40000}{v} - 16v$		
	$1200v\frac{\mathrm{d}v}{\mathrm{d}t} = 40000 - 16v^2$	M1	
	$75v\frac{\mathrm{d}v}{\mathrm{d}t} = 2500 - v^2$	A1	**<u>Given Answer</u>**
		(6)	
(b)	$75\int \frac{v}{2500 - v^2}\,\mathrm{d}v = \int \mathrm{d}t$	M1	Separate and attempt integration
	$-\frac{75}{2}\ln\left(2500 - v^2\right) = t \quad (+c)$	A1	
	$t = 0 \;\; v = 0 \;\Rightarrow -\frac{75}{2}\ln 2500 = c$	M1	Use initial values to find c
	$-\frac{75}{2}\ln\left(\frac{2500 - v^2}{2500}\right) = t$	A1	Or equivalent
	$\frac{2500 - v^2}{2500} = \mathrm{e}^{-\frac{2t}{75}} \;\rightarrow\; v^2 = 2500\left(1 - \mathrm{e}^{-\frac{2t}{75}}\right)$	M1	Find v or v^2 in terms of t
	$v = 50\sqrt{1 - \mathrm{e}^{-\frac{2t}{75}}}$	A1	
		(6) **[12]**	

Question Number	Scheme	Marks	
6.			
(a)	Length of string = $2\times 3l\sin\theta$ Extension = $6l\sin\theta - l$ E.P.E. = $\frac{4mg}{2l}(6l\sin\theta - l)^2$ G.P.E. of rod = $4mg\times 2l\cos 2\theta$ G.P.E. of mass at B = $kmg\times 4l\cos 2\theta$	B1	
	$V = \frac{4mg}{2l}(6l\sin\theta - l)^2 + 8mgl\cos 2\theta + 4kmgl\cos 2\theta +$ const	M1 A2	EPE term needs to be dimensionally correct. Need all three terms. Correct unsimplified
	$V = \frac{4mg}{2l}(6l\sin\theta - l)^2 + 8mgl(1-2\sin^2\theta) + 4kmgl\cos 2\theta + \text{const}$	M1	All in $\sin\theta$
	$= 2mgl(36\sin^2\theta - 12\sin\theta - 8\sin^2\theta - 4k\sin^2\theta) +$ const $= 8mgl((7-k)\sin^2\theta - 3\sin\theta) +$ constant	A1 **(6)**	**Given Answer**

Question Number	Scheme	Marks	
(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 8mgl\left(2(7-k)\sin\theta\cos\theta - 3\cos\theta\right)$	M1	Differentiate
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 \quad \left(2(7-k)\sin\theta - 3\right)\cos\theta = 0$	M1	Set derivative = 0
	$\sin\theta = \frac{3}{2(7-k)} \quad$ (or $\cos\theta = 0$, need not be seen)	A1	
	$\theta \leqslant \frac{\pi}{6} \quad \Rightarrow \frac{3}{2(7-k)} \leqslant \frac{1}{2}$	M1	Use of $\sin\theta \leq \frac{1}{2}$
	$3 \leqslant 7-k \quad k \leqslant 4 \quad$ *	A1 **(5)**	
(c)	$k = 4 \Rightarrow \theta = \frac{\pi}{6}$	B1	
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 8mgl\left[6\cos^2\theta - (6\sin\theta - 3)\sin\theta\right]$	M1	Second derivative ($8mgl$ or $24mgl$ not needed) [or differentiate $8mgl(3\sin 2\theta - 3\cos\theta)$]
	$= 8mgl\left[6\times\left(\frac{\sqrt{3}}{2}\right)^2 - 6\times\left(\frac{1}{2}\right)^2 + 3\times\frac{1}{2}\right]$	A1	Numerical unsimplified
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} > 0$	M1	by numerical evaluation or justification from trig terms $(36mgl)$
	V is min. $\quad\therefore$ stable equilibrium	A1	CSO
		(5) **[16]**	

Question Number	Scheme	Marks	
7			
(a)	In equilibrium $T = 0.5g = \frac{2.7e}{0.6}$	M1	
	$e = \frac{g}{9} = \frac{9.8}{9} = \frac{49}{45}$	A1	
	$0.6 + \frac{49}{45} - 4\sin 2t + y = 0.6 + x$		
	$y + \frac{49}{45} = x + 4\sin 2t$	A1	**Given Answer –** must see justification
		(3)	
(b)	$0.5g - \frac{2.7x}{0.6} = 0.5\ddot{y}$	M1A1	Equation of motion for P
	$g - 9x = \ddot{y}$		
	$g - 9\left(y + \frac{g}{9} - 4\sin 2t\right) = \ddot{y}$	DM1 A1	Substitute for x
	$\ddot{y} + 9y = 36\sin 2t$	A1	**Given Answer**
		(5)	

Question Number	Scheme	Marks	
(c)	C.F. is $y = A\cos 3t + B\sin 3t$	M1	
	Gen. soln. is $y = A\cos 3t + B\sin 3t + \frac{36}{5}\sin 2t$	A1	
	$t = 0 \quad y = 0 \quad \Rightarrow A = 0$	B1	
	$\dot{y} = 3B\cos 3t + \frac{72}{5}\cos 2t$	M1	Independent. Differentiate and use initial conditions to find B
	$t = 0 \quad \dot{y} = 0 \quad \Rightarrow 3B = -\frac{72}{5} \qquad B = -\frac{24}{5}$		
	$\therefore y = -\frac{24}{5}\sin 3t + \frac{36}{5}\sin 2t$	A1	
		(5)	
(d)	$\dot{y} = -\frac{72}{5}\cos 3t + \frac{72}{5}\cos 2t$	M1A1	
	$\dot{y} = -\frac{72}{5}\cos \pi + \frac{72}{5}\cos \frac{2}{3}\pi$	M1	Substitute $t = \frac{\pi}{3}$ in derivative to find $\dot{y}$
	$\dot{y} = 7.2$	A1	Final answer
		(4)	
		[17]	

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Summer 2013

GCE Mechanics 4 (6680/01)

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Summer 2013
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
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- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

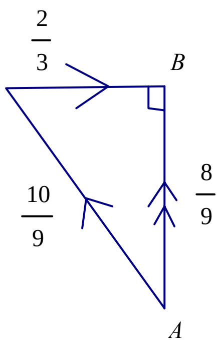
General Rules for Marking Mechanics

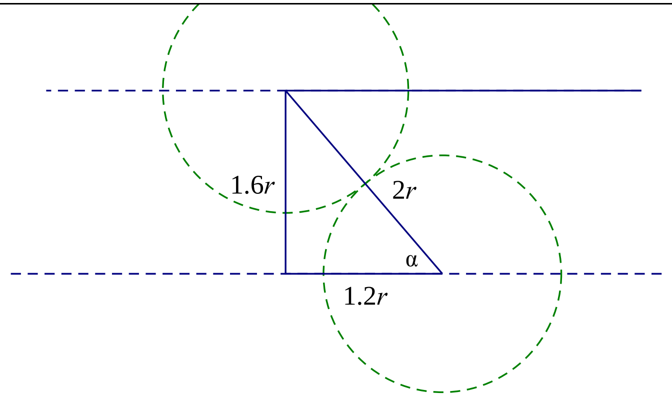
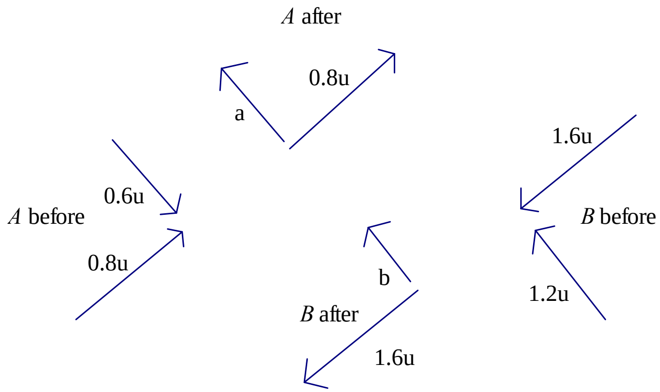
- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is accuracy error not method error.
- Omission of mass from a resolution is method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.

Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

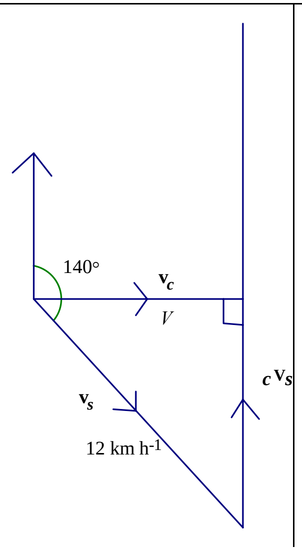
Question Number	Scheme	Marks	
1(a)	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1	Differential equation. All 3 terms required but condone sign errors
	NB: these two marks are available in (b) if not scored in (a)		
	$\int 1\mathrm{d}t=\int\frac{1}{9.8-3v}\mathrm{d}v$ $t+(C)=-\frac{1}{3}\ln(9.8-3v)$ $t=0,\ v=0\ \Rightarrow C=-\frac{1}{3}\ln 9.8$ $t=-\frac{1}{3}\ln\left(\frac{9.8-3v}{9.8}\right)$ $3v=9.8\left(1-e^{-3t}\right)$ ***Given Answer***	M1 A1=A1 M1 A1 A1 (8)	Separate the variables and attempt to integrate A1 for each side. C not needed Use initial conditions to evaluate Cor limits on a definite integral. Or equivalent **Watch out.** cwo
(a) alt	Equation of motion: $\frac{1}{2}g-\frac{3}{2}v=\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}$ $e^{3t}\frac{\mathrm{d}v}{\mathrm{d}t}+3e^{3t}v=ge^{3t},\ \frac{\mathrm{d}}{\mathrm{d}t}\left(ve^{3t}\right)=ge^{3t}$ $ve^{3t}=\frac{1}{3}ge^{3t}\left(+c\right)$ $t=0, v=0\Rightarrow 0=\frac{1}{3}g+C$ $\Rightarrow ve^{3t}=\frac{1}{3}g\left(e^{3t}-1\right),\ 3v=9.8\left(1-e^{-3t}\right)$	M1 A1 M1 A1=A1 M1 A1 A1	All 3 terms required but condone sign errors Use of integrating factor e^{3t} A1 for each side. $+C$ not required. Use initial conditions to evaluate C Correct equation in any equivalent form **Given form** cwo

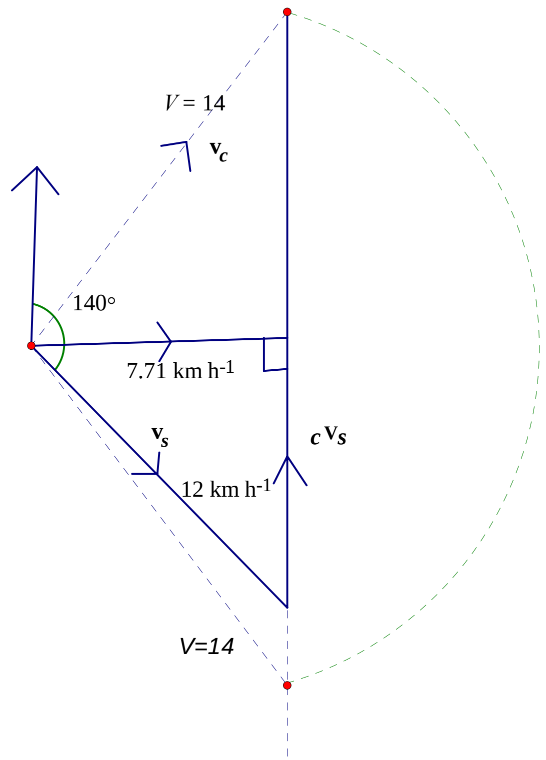
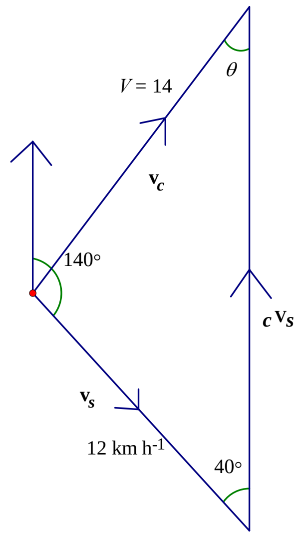
Question Number	Scheme	Marks	
1(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{9.8}{3}\left(1-e^{-3t}\right) \Rightarrow x=\frac{9.8}{3}\left(t+\frac{1}{3}e^{-3t}\right)(+C)$	M1 A1	Integrate the given v to find x C not needed
	$t=0, x=0 \Rightarrow C=-\frac{9.8}{9}$	M1 A1	Use the initial conditions to evaluate C or use limits correctly in a definite integral
	$t=2, x\approx 5.4$ (m)	A1 (5) **(13)**	5.45, $\frac{g}{9}\left(5+e^{-6}\right)$ or equivalent
(b) alt	$g-3v=v\frac{\mathrm{d}v}{\mathrm{d}x}$		
	$\int 1\mathrm{d}x=\int\frac{v}{g-3v}\mathrm{d}v=\int -\frac{1}{3}+\frac{g}{3(g-3v)}\mathrm{d}v$	M1	Separate the variables and rearrange the RHS
	$x=-\frac{v}{3}-\frac{g}{9}\ln(g-3v)+C$	A1	$+C$ not needed
	$x=0, v=0 \Rightarrow C=\frac{g}{9}\ln g$ and $t=2, v=\frac{g}{3}\left(1-e^{-6}\right)(=3.258....)$	M1 A1	Use the initial conditions to find C & find the value of v when $t=2$
	$x=\frac{g}{9}\left(1-e^{-6}\right)-\frac{g}{9}\ln\left(e^{-6}\right)=5.4$	A1	
		(5)	
		(13)	

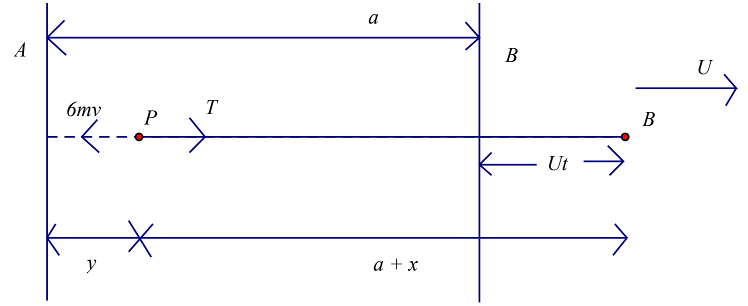
Question Number	Scheme	Marks	
2(a)	Shortest time $50 \div \frac{10}{9}$,= 45 (s)	M1,A1	
(b)	Drifts $\frac{2}{3} \times "45"$, = 30 (m)	M1 A1	$\frac{2}{3} \times$ their time
(c)	 Trig or pythag to find velocity of swimmer in direction *AB* $\frac{8}{9}$ ms^{-1} $50 \div "\frac{8}{9}", = 56.25$ (s)	M1 A1 DM1,A1 **(8)**	 0.88 or better Dependent on the previous M 56 or better

Question Number	Scheme	Marks	
3			
	$0.6u$ or $u\cos\alpha$	B1	component of the initial velocity of A parallel to the line of centres on impact
	$1.2u$ or $2u\cos\alpha$	B1	component of the initial velocity of B parallel to the line of centres on impact
	$2m\times1.2u-3m\times0.6u=3ma+2mb$	M1	CLM parallel to the line of centres. Requires all the terms.
	$(3a+2b=0.6u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$
	$e(1.2u+0.6u)=a-b$	M1	Restitution parallel to the line of centres. Must be used the right way round.
	$(a-b=0.3u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$ If signs are inconsistent between the two equations, penalise here.
		DM1	Solve a pair of simultaneous eqns in a & b for one of a & b. Dependent on the two previous M marks.
	a = 0.24u or b = -0.06u	A1	In terms of u only
	$(1.2u-(-0.06u))\times2m=2.52mu$	M1	Find impulse on A or B. Unsimplified. For their a or b. Correct mass for the velocities used.
	or $(0.24u-(-0.6u))\times3m=2.52mu$	A1 **(10)**	$\frac{63}{25}$

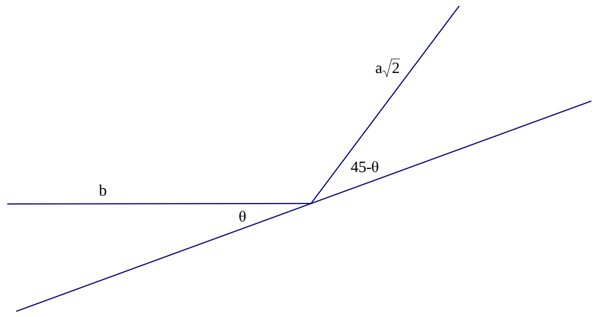
Question Number	Scheme	Marks	
4(a)	PE of ring = $-mgx$	B1	Taking the level of the peg as zero PE
	PE of particle = $-3mg(L-\sqrt{x^2+d^2})$	M1 A1	
	$\Rightarrow V = 3mg\sqrt{x^2+d^2} - mgx$ + constant. **AG**	A1 (4)	Watch out
(b)	$\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{3mg.2x}{2\sqrt{x^2+d^2}} - mg$	M1	
	$\frac{dV}{dx} = 0 \Rightarrow 3x = \sqrt{x^2+d^2}$, $9x^2 = x^2+d^2$, $8x^2 = d^2$	M1	Set $\frac{dV}{dx} = 0$ and solve for x
	$x = \frac{d}{\sqrt{8}} = \left(\frac{\sqrt{2}d}{4}\right)$	A1	0.354d of better
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = 3mg\left(\frac{\sqrt{x^2+d^2}.1 - x.\frac{2x}{2\sqrt{x^2+d^2}}}{x^2+d^2}\right) =$	M1	Product or quotient rule $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = \frac{3mg}{\sqrt{x^2+d^2}} - \frac{3mgx}{2}.2x.\left(x^2+d^2\right)^{-\frac{3}{2}}$
	$3mg\left(\frac{\sqrt{9x^2}.1 - x.\frac{2x}{2\sqrt{9x^2}}}{9x^2}\right) = \frac{3mgd^2}{\left(x^2+d^2\right)^{\frac{3}{2}}}$ (>0)	A1	OR $= 3mg\left(\frac{3x-\frac{x}{3}}{9x^2}\right)(>0)$ Correct unsimplified. $\frac{16\sqrt{2}mg}{9d}$, $2.5\frac{mg}{d}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = \frac{9mgd}{\sqrt{8}}$
	Stable	A1ft **(10)**	Correct conclusion for their expression

Question Number	Scheme		Marks	
5(a)		Minimum $V = 12\cos 50°$ ≈ 7.71	M1 A1 A1	Use of triangle with right angle between v_C and $_Cv_S$. Condone sin/cos confusion. Correct unsimplified trig expression 7.71 only

Question Number	Scheme	Marks	
5(b)			
	Vector triangle for relative velocities when $V = 14$	M1	could have relative velocity due S. Could show both possibilities.
	Select the vector triangle with the relative velocity due N.	A1	
	$\frac{\sin\theta}{12} = \frac{\sin 40}{14}$	DM1 A1	Use of sine rule or equivalent to find θ
	Bearing 033°	A1 **(8)**	Final answer. Accept 33.4°

Question Number	Scheme	Marks	
6(a)			
	$a + Ut = y + (a + x)$	M1	Diagram or clear explanation using distances
	$Ut = x + y$ ***Answer Given***	A1	Watch out for fudges.
(b)	$T = \dfrac{9ma \times x}{a} = 9mx$	B1	
	$T - 6m\dot{y} = m\ddot{y}$	M1	Equation of motion of P. Requires all 3 terms in terms of x and/or y
	$9mx - 6m(U - \dot{x}) = -m\ddot{x}$	A2	Expressed in terms of x. -1 each error
	$\ddot{x} + 6\dot{x} + 9x = 6U$	A1	**Answer given**. Watch out for fudges
(c)	$t = 0, x = 0, \dot{x} = U \quad 0 = AU + \dfrac{2U}{3}, \; A = -\dfrac{2}{3}$	M1 A1	Use initial conditions to find A
	$\dot{x} = BUe^{-3t} - 3(A + Bt)Ue^{-3t}$	M1 A1	Differentiate
	$U = BU - 3AU, \; B = 3A + 1 = -1$	A1	
(d)	$\dot{y} = U - \dot{x} = U - \left(-Ue^{-3t} + 2Ue^{-3t} + 3Ute^{-3t}\right)$	M1	
	$= U\left(1 - e^{-3t} - 3te^{-3t}\right)$	A1	Or equivalent
		(14)	

Question Number	Scheme	Marks	
7(a)	State that impulse acts perpendicular to the wall and demonstrate that (2**i** + **j**).(-**i** +2**j**) = 0	B1	Requires scalar product or gradient diagram.
(b)	Impulse momentum equation: $m(\mathbf{v} - \mathbf{u}) = m[(a-b)\mathbf{i} + a\mathbf{j}] = \lambda(-\mathbf{i} + 2\mathbf{j})$ $\Rightarrow a = -2(a-b)$, $3a = 2b$ OR Taking scalar products of velocities with (2**i** + **j**) $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3a$ No change parallel to the wall so $2b = 3a$. Scalar products with (–**i** +2**j**): $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = a$ Impact equation: $a=eb$ $e = \frac{2}{3}$	M1 A2 A1 M1 A1A1 A1 B1 M1A1 A1	Requires all terms present and of the correct structure -1 each error

Question Number	Scheme	Marks	
7(b) alt			
	$b\cos\theta = a\sqrt{2}\cos(45-\theta)$	M1 A2	Parallel to the wall. Condone trig confusion? -1 each error. Both angles in same variable?
	$b\cos\theta = a\cos\theta + a\sin\theta,\ \ 2b-2a=a$ $2b=3a$	A1	
	Use of $\tan\theta = \frac{1}{2}$	B1	When seen in (b). Implied by 26.6 or 18.4
	$a\sqrt{2}\sin(45-\theta) = eb\sin\theta$	M1	Perpendicular to the wall. Condone consistent trig confusion?
	$a\cos\theta = (a+eb)\sin\theta,\ \ 2a = a+eb$	A1	$e=\sqrt{\frac{10a^2}{b^2}-4}$
	$e=\frac{2}{3}$	A1	0.67 or better
(c)	Fraction of KE lost $=\frac{b^2-2a^2}{b^2}$	M1A1	
	$=\frac{1-2\times\frac{4}{9}}{1}=\frac{1}{9}$	A1	
		(12)	

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Summer 2014

Pearson Edexcel GCE in Mechanics 4R (6680/01R)

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PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

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These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

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These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

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These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

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(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

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- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

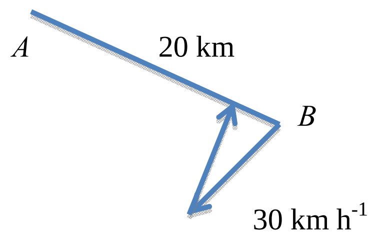
  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme		Marks
1.	$v\cos 45^{\circ} = u\sin\alpha$	parallel	M1 A1
	$v\sin 45^{\circ} = eu\cos\alpha$	perpendicular	M1 A1
	$e = \tan\alpha$	or square & add	M1 A1
	$I = m(v\cos 45^{\circ} + u\cos\alpha)$	impulse	M1 A1
	$= mu(\sin\alpha + \cos\alpha)$	in terms of $u,\ \alpha$	M1
	$= \dfrac{mu(1+e)}{\sqrt{1+e^2}}$	in terms of $u,\ e$	M1 A1
			11

Question Number	Scheme	Marks
2. (a)	 Vector triangle with perpendicular	M1
	Minimum speed = 30sin60°	M1
	= 15√3 oe (26.0)	A1 (3)
(b)	$\frac{\sin\theta}{30}=\frac{\sin 60°}{32}$	M1 A1
	$\Rightarrow \theta = 54.281....°$	A1
	$v^2 = 32^2 + 30^2 - 2.32.30\cos(120° - \theta)$	M1 A1
	$= 33.68....$ $\left(15+\sqrt{349}\right)$	A1
	$t = \frac{20}{33.68...} = 0.59379...\text{h} = 35.62...\text{min}$	M1
	Time is 2.36 pm (14.36)	A1 (8)
		11

Question Number	Scheme		Marks
3.	$-(mg + mkv) = mv\dfrac{\mathrm{d}v}{\mathrm{d}x}$	Differential equation	M1 A1
	$\int_0^H \mathrm{d}x = \int_U^0 -\dfrac{v}{(g+kv)}\mathrm{d}v$	Separate variables	M1
	$\int_0^H \mathrm{d}x = -\int_U^0 \dfrac{1}{k} - \dfrac{g}{k(g+kv)}\mathrm{d}v$	Split for integration	M1 A1
	$-\quad H = \left[\dfrac{-v}{k} + \dfrac{g}{k^2}\ln(g+kv)\right]_U^0$		A1
	$= \dfrac{U}{k} - \dfrac{g}{k^2}\ln\left(\dfrac{g+kU}{g}\right)$	Use of limits	M1 A1
			8

Question Number	Scheme		Marks
4. (a)	$mv_1 + mv_2 = mu\cos 60^\circ$	Momentum	M1 A1
	$-v_1 + v_2 = eu\cos 60^\circ$	Impact law	M1 A1
	$v_1 = \frac{u(1-e)}{4}$		
	Speed of $S = \sqrt{\frac{u^2(1-e)^2}{16} + \frac{3u^2}{4}}$	Solve for v_1 and find speed	M1 A1
	$= \frac{u}{4}\sqrt{e^2 - 2e + 13}$		A1
	$\tan\theta = \frac{u\sqrt{3}}{2v_1} = \frac{2\sqrt{3}}{(1-e)}$	Use components to find dirn	M1 A1
	S moves at $\arctan\frac{2\sqrt{3}}{(1-e)}$ to the line of centres		M1 A1
	$v_2 = \frac{u(1+e)}{4}$	v_2 in terms of u, e	B1 (12)
(b)	T has speed $\frac{u(1+e)}{4}$ along the line of centres	Conclusion	
	θ is a max when $e = 1$ then $\theta = 90^\circ$		M1 A1
	then deflection angle is $90^\circ - 60^\circ = 30^\circ$ $\delta = 30$		A1 (3) **15**

Question Number	Scheme		Marks
5.(a)	$-12mgl\sin\theta$	rod	B1
	$-mg(L-4l\sin\frac{1}{2}\theta)$	particle	M1 A1
	$4mgl(\sin\frac{1}{2}\theta-3\sin\theta)+\text{constant}$	*given answer*	A1 (4)
(b)	$\frac{dV}{d\theta}=4mgl(\frac{1}{2}\cos\frac{1}{2}\theta-3\cos\theta)$	Differentiate	M1 A1
	$4mgl(\frac{1}{2}\cos\frac{1}{2}\theta-3\cos\theta)=0$	Derivative = 0	M1
	$\frac{1}{2}\cos\frac{1}{2}\theta-3(2\cos^2\frac{1}{2}\theta-1)=0$	In terms of $\cos\frac{1}{2}\theta$	M1
	$\cos\frac{1}{2}\theta=\frac{3}{4}$ or $-\frac{2}{3}$		A1
	$\cos\theta=\frac{1}{8}$ or $-\frac{1}{9}$	Solve for θ	M1 A1
	$\theta=1.45$ as $\theta<\frac{1}{2}\pi$ (83.0°)		A1
	$\frac{d^2V}{d\theta^2}=4mgl(-\frac{1}{4}\sin\frac{1}{2}\theta+3\sin\theta)$	Second derivative	M1
	When $\theta=1.45$, $\frac{d^2V}{d\theta^2}=11.25mgl>0$, hence stable		A1 (10)
			14

Question Number	Scheme		Marks
6.(a)	$-Mkv - T = M\ddot{x}$		M1 A1
	$-Mk\dot{x} - \frac{Mn^2ax}{a} = M\ddot{x}$	In terms of x	M1
	$\ddot{x} + k\dot{x} + n^2x = 0$ *Answer Given*		A1 (4)
(b)	Aux equn: $p^2 + \frac{5n}{2}p + n^2 = 0$		M1
	$p = -\frac{n}{2}$ or $-2n$		
	$x = Ae^{-\frac{n}{2}t} + Be^{-2nt}$	General solution	A1
	$t = 0, x = 0 \Rightarrow 0 = A + B$		B1
	$\dot{x} = -\frac{nA}{2}e^{-\frac{n}{2}t} - 2nBe^{-2nt}$	Differentiate	M1
	$t = 0, x = U \Rightarrow U = -\frac{nA}{2} - 2nB$		A1
	$\Rightarrow -\frac{2U}{n} = A + 4B$		
	$A = \frac{2U}{3n}; B = -\frac{2U}{3n}$	Solve for A, B	M1
	$x = \frac{2U}{3n}e^{-\frac{n}{2}t} - \frac{2U}{3n}e^{-2nt}$		A1 (7)
(c)	$\dot{x} = \frac{U}{3}(4e^{-2nt} - e^{-\frac{n}{2}t})$		M1
	$\dot{x} = 0 \Rightarrow 4e^{-2nt} - e^{-\frac{n}{2}t} = 0$		M1
	$e^{\frac{3n}{2}t} = 4$		A1
	$x = \frac{2U}{3n}(2^{-\frac{2}{3}} - 2^{-\frac{8}{3}}) = \frac{U}{n}(2^{-\frac{5}{3}})$ $\left(0.31\frac{U}{n}\right)$		M1 A1 (5)
			16

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics M4 (6680/01)

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Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

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Summer 2014
Publications Code UA039503

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
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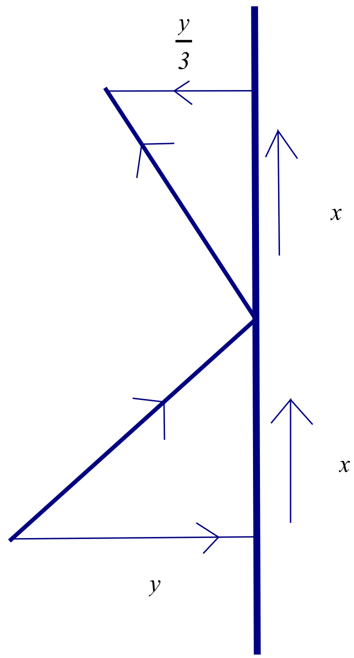
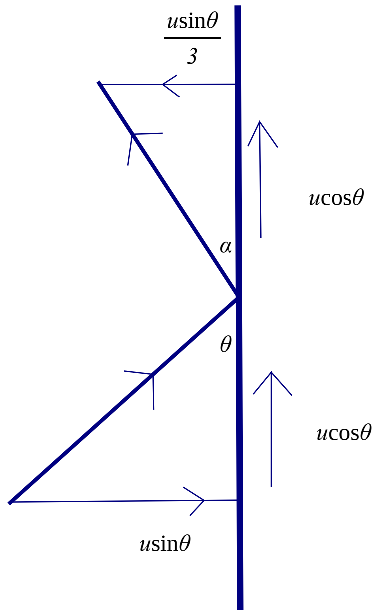
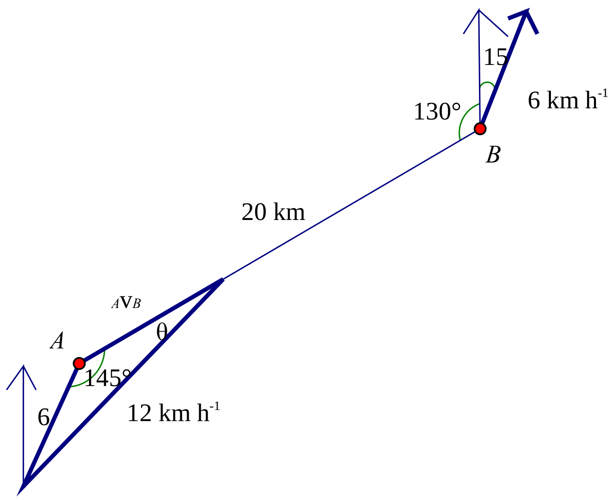
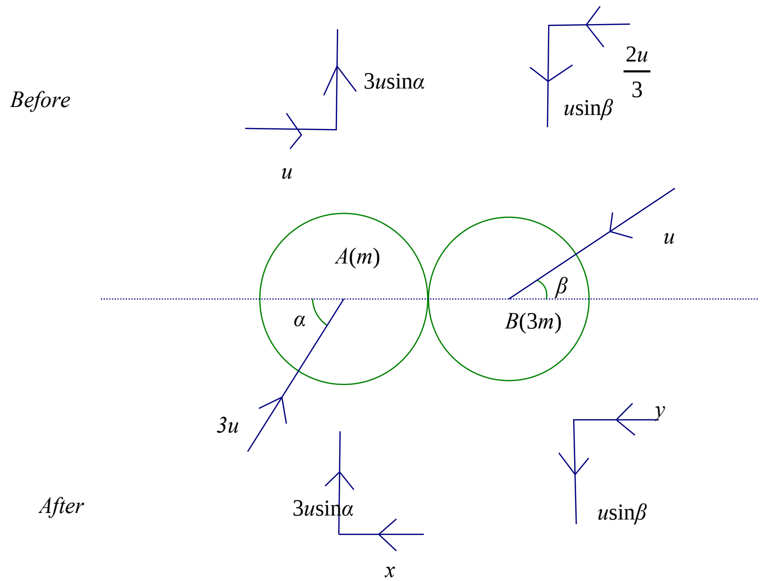
1a	$\mathbf{r}_A = (-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) + t\,(3\mathbf{i} + \mathbf{j}) = ((-6 + 3t)\mathbf{i} + (4 + t)\mathbf{j} + (-3)\mathbf{k})$	M1	Position vector for A or B
	$\mathbf{r}_B = (-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(\mathbf{i} - \mathbf{k}) = ((-2 + t)\mathbf{i} + (2)\mathbf{j} + (3 - t)\mathbf{k})$	A1	Both position vectors correct (seen or implied)
	${}_B\mathbf{r}_A = (-2 + \mathrm{t} + 6 - 3t)\mathbf{i} + (2 - 4 - t)\mathbf{j} + (3 - t + 3)\mathbf{k}$ $= (4 - 2t)\mathbf{i} + (-2 - t)\mathbf{j} + (6 - t)\mathbf{k}$	M1	Position of B relative to A (or A relative to B)
	$\left\|{}_B\mathbf{r}_A\right\|^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2$	M1	Use of Pythagoras
alt1	$= 6t^2 - 24t + 56 = 6(t-2)^2 + 32$	M1	Complete the square
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1 [6]	Reach **given answer** correctly
	$\left\|{}_B\mathbf{r}_A\right\|^2 = (4-2t)^2 + (t+2)^2 + (6-t)^2\ \left(= 6t^2 - 24t + 56\right)$	M1	Use of Pythagoras
alt2	$12t - 24 = 0 \Rightarrow t = 2$	M1	Differentiate and solve for t
	Minimum distance = $\sqrt{32} = 4\sqrt{2}$ m **	A1	Reach **given answer** correctly
alt3	$\begin{pmatrix} 4-2t \\ -2-t \\ 6-t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow 8 - 4t - 2 - t + 6 - t = 12 - 6t = 0$	M1	Scalar product of position vector with relative velocity = zero and form equation in t
	Distance $= \sqrt{0^2 + 4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$	M1	Use of Pythagoras
		A1	Reach **given answer** correctly
1b	When $t = 2$,	B1	Seen or implied
	$\mathbf{r}_A = 6\mathbf{j} - 3\mathbf{k}$	B1 [2]	cso

Question Number	Scheme	Marks	Notes
2a	$\frac{P}{v}-10v=ma\,;\frac{25000}{v}-10v=1000a$	M1	Equation of motion
	v = 20, (m s^{-2}) $a=\dfrac{\frac{25000}{20}-10\times 20}{1000}=\dfrac{\frac{25}{2}-2}{10}$	DM1	Substitute $v=20$
	$=1.05$ (m s^{-2}) **	A1 [3]	Obtain **given answer** correctly
2b	$v\frac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{\frac{25000}{v}-10v}{1000}=\dfrac{25000-10v^2}{1000v}=\dfrac{2500-v^2}{100v}$	M1 A1	Differential equation in v and x Any equivalent form
	$\int\frac{100v^2}{2500-v^2}\mathrm{d}v=\int 1\mathrm{d}x\qquad\left(=100\int -1+\frac{2500}{2500-v^2}\mathrm{d}v\right)$	M1	Separate the variables
alt1	$=100\int -1+\frac{25}{50-v}+\frac{25}{50+v}dv$	DM1 A1	Split using partial fractions Or equivalent
	$x(+C)=100\left\{-v+25\ln\left\lvert\frac{50+v}{50-v}\right\rvert\right\}$	A1	Integration correct
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\mathrm{m})$	DM1 A1 [8]	Correct use of limits Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
alt2	$=100\left(v-50\operatorname{arc\,tanh}\left(\frac{v}{50}\right)\right)$	DM1 A1	Use of arctanh correct
	$x(+C)=100\left\{-v+25\ln\left\lvert\frac{50+v}{50-v}\right\rvert\right\}$	A1	Convert to log form
	$x=100\left(-20+25\ln\frac{70}{30}\right)-100\left(-10+25\ln\frac{60}{40}\right)=105\,(\mathrm{m})$	DM1 A1	Correct use of limits Or better $\left(2500\ln\left(\frac{14}{9}\right)-1000\right)$
	NB A correct numerical answer that does not follow from integration scores no marks.		

Question Number	Scheme	Marks	Notes
3 alt1			
	Speed perpendicular to wall after collision = $\frac{y}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{1}{2}\left(x^2+y^2\right)=x^2+\frac{1}{9}y^2$	M1 A1	Use the speeds to form an equation in x & y (or equivalent) Correct unsimplified
	$9\left(x^2+y^2\right)=2\left(9x^2+y^2\right),\ \ 9x^2=7y^2,\ \ x=\frac{\sqrt{7}}{3}y$	A1	Correct ratio for x & y (any equivalent form)
	direction deflected by $\tan^{-1}\frac{y}{x}+\tan^{-1}\frac{y}{3x}$	M1 A1	To find the correct angle Correct in x & y
	$=\tan^{-1}\sqrt{\frac{27}{5}}+\tan^{-1}\sqrt{\frac{3}{5}}=104.5°$ (104)	A1 [8]	

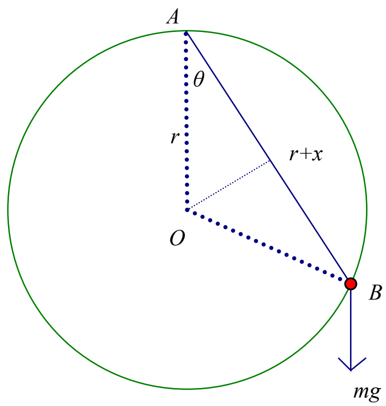
Question Number	Scheme	Marks	Notes
alt2			
	Speed perpendicular to wall after collision = $\frac{u\sin\theta}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{u^2}{4}=\frac{u^2}{9}\sin^2\theta+u^2\cos^2\theta$	M1 A1	Use the speeds to form an equation in $u\ \&\ \theta$ (or equivalent) Correct unsimplified
	$27\cos^2\theta=5\sin^2\theta,\ \ \tan^2\theta=\frac{27}{5}$	A1	Correct trig ratio for θ (or equivalent)
	deflected by $\theta+\alpha$, $\tan(\theta+\alpha)=\dfrac{\tan\theta+\frac{1}{3}\tan\theta}{1-\frac{1}{3}\tan^2\theta}$ $\left(=-\sqrt{15}\right)$	M1 A1	To find the correct angle Correct in θ (or equivalent)
	$\theta+\alpha=104.5°$ (104)	A1 [8]	

Question Number	Scheme	Marks	Notes
4a			
	Relative velocity triangle	M1	Seen or implied
	$\frac{\sin 145}{12} = \frac{\sin\theta}{6}$, $\theta = 16.7°$	M1	Use of trig to find a relevant angle
	Bearing = $15 + (180 - 145 - 16.7) = 33.3°$	M1	To find the required angle
	Bearing 033°	A1 [4]	They were asked for an answer "to the nearest degree". Accept N 33° E
4b	$\frac{{}_Av_B}{\sin 18.3} = \frac{12}{\sin 145}$	M1	Correct method to find the relative velocity
	${}_Av_B = 6.58$ (km h^{-1})	A1	
	Time taken = $\frac{20}{6.58}$ (hrs)	M1	For their 6.58
	Time is 3:02 pm (1502)	A1 [4]	

Question Number	Scheme	Marks	Notes
5a			
	CLM: $mx+3my=3m\times u\cos\beta-m\times 3u\cos\alpha=mu\ \ (x+3y=u)$	M1 A1	Terms of correct structure but condone sign errors
	NEL: $x-y=\frac{1}{5}(3u\cos\alpha+u\cos\beta)\left(=\frac{1}{5}\left(u+\frac{2}{3}u\right)=\frac{1}{3}u\right)$	M1 A1	equation of correct structure but condone sign errors
	$x=\frac{u}{2}$, or $y=\frac{u}{6}$	DM1 A1	Dependent on the two previous M marks. Solve for x or y Correct for their x or y
	Magnitude of the impulse on A = $mu-\left(m\times-\frac{u}{2}\right)=\frac{3mu}{2}$	M1 A1 [8]	Must be positive

Question Number	Scheme	Marks	Notes
5b	Component of velocity perpendicular to the line of centres before = component after = $3u\sin\alpha = 3u\times\frac{\sqrt{8}}{3} = \sqrt{8}u$	B1	
	KE lost = $\frac{m}{2}\left(9u^2 - \left(8u^2 + \frac{1}{4}u^2\right)\right)\left[= \frac{3}{8}mu^2\right]$	M1	Change in KE. Does not need to be a fraction at this stage. Does not need to include the (cancelling) component perpendicular to the line of centre.
		A1	Correct unsimplified
	Fraction lost = $\frac{3/8}{9/2} = \frac{3}{8}\times\frac{2}{9} = \frac{1}{12}$	A1 [4]	

Question Number	Scheme	Marks	Notes
6a	$m\ddot{x} = 4mv - \frac{5ma \times x}{a}$ $\quad v = -\dot{x}$	M1	Equation of motion as far as $m\ddot{x} = \pm 4mv - T$
		M1	Use of $v = -\dot{x}$
	$\ddot{x} + 4\dot{x} + 5x = 0$ **	A1 [3]	Reach **given answer** correctly.
6b	AE $m^2 + 4m + 5 = 0$, $m = \frac{-4 \pm \sqrt{4^2 - 4\times 5}}{2} = -2 \pm \mathrm{i}$	M1	Solve AE to find GS
	$x = \mathrm{e}^{-2t}(A\cos t + B\sin t)$	A1	
	$t = 0, x = a = A$	M1 A1	Use $t = 0, x = a$ to find A
	$\dot{x} = -2\mathrm{e}^{-2t}(a\cos t + B\sin t) + \mathrm{e}^{-2t}(-a\sin t + B\cos t)$	M1	Differentiate and use boundary conditions to find B
	$t = 0$, $\quad \dot{x} = 0 = -2a + B$ $\quad x = \mathrm{e}^{-2t}(a\cos t + 2a\sin t)$	A1 [6]	
6c	String goes slack when $x = \mathrm{e}^{-2t}(a\cos t + 2a\sin t) = 0$		
	$\cos t = -2\sin t$, $\tan t = -\frac{1}{2}$	M1 A1	Set $x = 0$ and solve for t or $\tan t$
	$\dot{x} = -2\mathrm{e}^{-2t}(a\cos t + 2a\sin t) + \mathrm{e}^{-2t}(-a\sin t + 2a\cos t)$	M1	Substitute a positive value of t to find the speed. An answer of 0.88... indicates a negative t.
	$= \mathrm{e}^{-2t}(-5a\sin t) = -0.01\ldots a$ $\quad$ Speed $= 0.011a$ (ms^{-1})	A1 [4]	The question specifies 2 sf

Question Number	Scheme	Marks	Notes
7a			
	Measuring GPE from A, GPE $= -mg\cos\theta(r+x)$	B1	Or $-2mgr\cos^2\theta$, or $-mgr(1+\cos 2\theta)$ or equivalent
	EPE $= \dfrac{kmgx^2}{2r}$	B1	
	From the isosceles triangle, $\cos\theta = \dfrac{x+r}{2r}$	B1	
	$V = -mg\cos\theta(r+x) + \dfrac{kmgx^2}{2r}$	M1	Correct unsimplified total
	$= -mg\cos\theta \times 2r\cos\theta + \dfrac{kmgr^2(2\cos\theta - 1)^2}{2r}$	A1	In terms of r & θ
	$= mgr\left\{-2\cos^2\theta + 2k\cos^2\theta - 2k\cos\theta + \dfrac{k}{2}\right\}$		
	$= 2mgr\{(k-1)\cos^2\theta - k\cos\theta\} + \text{constant}$ **	A1 (6)	Reach **given answer** correctly

Question Number	Scheme	Marks	Notes
7b	$V=2mgr\left(2\cos^2\theta-3\cos\theta\right)$+constant		
	$V'=2mgr\left(-4\cos\theta\sin\theta+3\sin\theta\right)$	M1 A1	Differentiate V
	$V'=0\Rightarrow\sin\theta=0$ or $\cos\theta=\dfrac{3}{4}$	M1	Derivative = 0 and solve for θ
	$\theta=0$ or $\theta=\pm0.72$ rads	A3	-1 for each missing solution
	$V''=2mgr\left(-4\cos2\theta+3\cos\theta\right)$	M1	Second derivative of V
	$\theta=0, V''=-2mgr<0$, unstable equilibrium	A1	Need to see $-2mgr$ or equivalent
	$\cos\theta=\dfrac{3}{4}$, $V''=\dfrac{7mgr}{2}>0$, stable equilibrium	A1 (9)	Do not need to consider the symmetrical position as well

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edexcel

Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Mechanics 4 (6680/01)

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Summer 2015
Publications Code UA042157

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

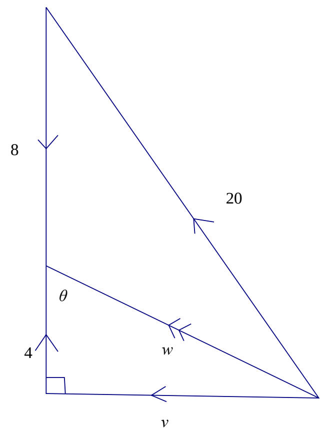
  RHS, LHS Right hand side, left hand side.

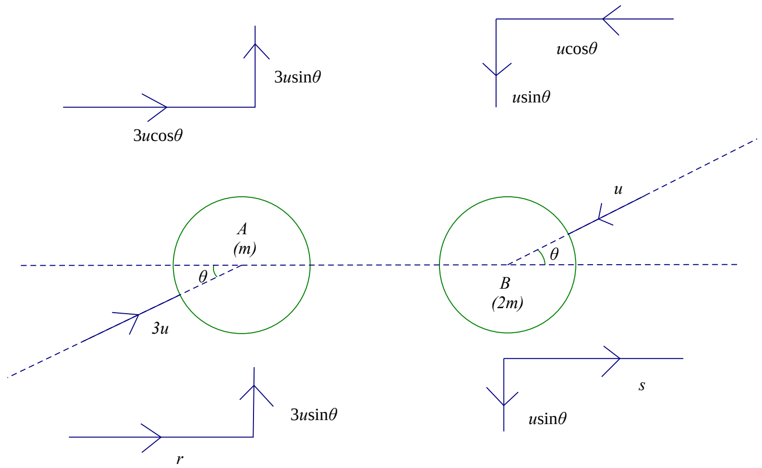
June 2015
6680 Mechanics 4
Mark Scheme

Question Number	Scheme	Marks	Notes
1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Find position vector of one particle relative to the other . $\mathbf{r}_P = \begin{pmatrix} 16+t \\ -12+2t \end{pmatrix}$, $\mathbf{r}_Q = \begin{pmatrix} -5+2t \\ 4+t \end{pmatrix}$
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2$		Pythagoras
	$\frac{\mathrm{d}}{\mathrm{d}t}d^2 = -2(21-t) + 2(-16+t)\ (= -74+4t)$	M1	Differentiate d or d^2 wrt t
		M1	Set derivative = 0 and solve for t
	Min when $t = 18.5(\text{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\frac{25}{2}} = 3.54$ (m)	A1	
		[7]	
	See over for alternatives.		

Question Number	Scheme	Marks	Notes
alt1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2 \; (= 2t^2 - 74t + 697)$	M1	Use Pythagoras to express d^2 as a quadratic in t
		M1	Complete the square
	$2(t-18.5)^2 - 684.5 + 697$	A1	
	Min $d^2 = 697 - 684.5$	M1	Use completed square to find minimum value for their expression
	Min. $d = \sqrt{697-684.5} = \sqrt{12.5}$	A1	
alt2	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
	$: \begin{pmatrix} 21-t \\ -16+t \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -(21-t) + (-16+t) = 0$,	M1	Set scalar product of relative position and relative velocity = 0 and solve for t.
	$t = 18.5\,(\text{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\dfrac{25}{2}} = 3.54$ (m)	A1	
	See over for alternative		

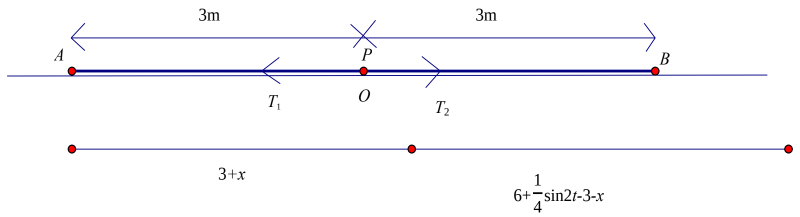
Question Number	Scheme	Marks	Notes
alt 3	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Initial position of P relative to Q
	$= \begin{pmatrix} 21 \\ -16 \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
		M1	Use scalar product to find $\cos\theta$
	$\cos\theta = \dfrac{-37}{\sqrt{2}\sqrt{687}}$ (-0.998....)	A1	Accept +/-
	$d = PQ\sin\theta$	M1	Use trig to find distance
	$= \sqrt{697} \times \sqrt{1 - \dfrac{37^2}{2 \times 697}} = \dfrac{5}{\sqrt{2}} \approx 3.54$	A1	
		[7]	

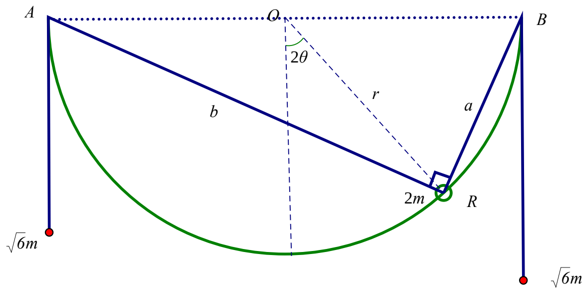
Question Number	Scheme	Marks	Notes
2		B1	Either triangle of velocities
		M1	Two triangles combined using their common velocity
		A1	Correct diagram seen or implied
	Correct method to obtain one of v, w, θ	M1	($v = 16, w = 16.5, \theta = 76°$) Make it dependent?
	speed is 16.5(km h^{-1})	A1	$4\sqrt{17}$
	Direction S 76° E or equivalent	A1	104°
		[6]	
Alt	Velocity of wind = w		
	$w = -v\mathbf{i} + 4\mathbf{j}$	B1	one correct equation
	$w = a\mathbf{i} + b\mathbf{j} - 8\mathbf{j}$ $\quad a^2 + b^2 = 400$	M1	2nd equation and compare coefficients
	coeff $\mathbf{j}$: $4 = b - 8$ $\quad b = 12$	A1	2 correct eqns
	$\mathbf{i}$: $-v = a$		
	$a^2 + 144 = 400 \Rightarrow a = -16 \quad (v > 0)$	M1	
	$\lvert w \rvert = \sqrt{4^2 + 16^2} = 4\sqrt{17}$	A1	
	Bearing 104°	A1	

Question Number	Scheme	Marks	Notes
3			
	After collision $u\sin\theta$ and $3u\sin\theta$ perpendicular to *l* of *c*	B1	
	CLM : $r+2s=3u\cos\theta-2u\cos\theta\left(=u\cos\theta\right)$	M1	Requires all four terms but condone sign errors.
		A1	Correct unsimplified equation
	Impact: $s-r=e\times 4u\cos\theta\left(=\dfrac{u\cos\theta}{2}\right)$	M1	Must be the right way round, but condone sign errors
		A1	Correct unsimplified equation
	$\Rightarrow r=0, s=\dfrac{u\cos\theta}{2}$	DM1	Solve the simultaneous equations to find the horizontal components of velocities. Dependent on the two preceding M marks
		A1	Both correct
	After the collision: $\left(3u\sin\theta\right)^2+r^2=4\left(\left(u\sin\theta\right)^2+s^2\right)$	M1	Use $v_A=2v_B$. Condone 2 on the wrong side?
		A1	Correct unsimplified equation (their *r* and *s*)
	$9u^2\sin^2\theta=4u^2\sin^2\theta+4.\dfrac{u^2}{4}\cos^2\theta$	A1	Obtain an equation in θ
	$\tan^2\theta=\dfrac{1}{5},\quad \theta=24.1(^\circ)\ \ (0.421 \text{ radians})$	DM1	Solve for θ. Dependent on the previous M1
		A1	Correct to 3 sf or better

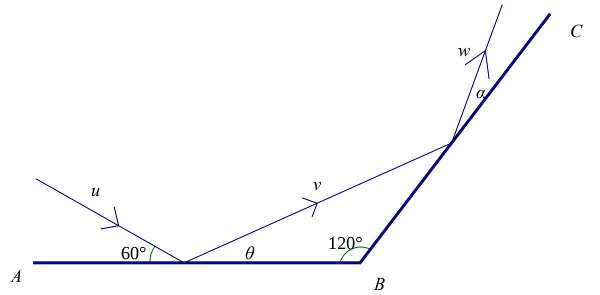
Question Number	Scheme	Marks	Notes
3 alt	For those who prefer everything with trig:		
	$v_A \sin\alpha = 3u\sin\theta$, $v_B \sin\beta = u\sin\theta$	B1	Perpendicular to the l.o.c.
	$m.3u\cos\theta - 2m.u\cos\theta = mv_A\cos\alpha + 2mv_B\cos\beta$	M1	CLM
	$\left(u\cos\theta = v_A\cos\alpha + 2v_B\cos\beta\right)$	A1	
	$\frac{1}{8}\times\left(3u\cos\theta + u\cos\theta\right) = v_B\cos\beta - v_A\cos\alpha$	M1	Impact law
	$\left(\frac{u}{2}\cos\theta = v_B\cos\beta - v_A\cos\alpha\right)$	A 1	
	$\frac{u}{2}\cos\theta = v_B\cos\beta$, $0 = v_A\cos\alpha \left(\Rightarrow \sin\alpha = 1\right)$	DM1	Simultaneous equations
		A1	
	$v_A\sin\alpha = v_A = 2v_B = 3u\sin\theta$	M1	Use $v_A = 2v_B$ to find β
	$v_B\sin\beta = u\sin\theta \Rightarrow \frac{3u\sin\theta}{2}\sin\beta = u\sin\theta$	A1	
	$\sin\beta = \frac{2}{3}$	A1	
	$2v_B = 3u\sin\theta$ & $\frac{u}{2}\cos\theta = v_B\cos\beta$ $\Rightarrow 6\tan\theta = \frac{2}{\cos\beta}\left(= 2\times\frac{3}{\sqrt{5}}\right)$	M1	Solve for θ
	$\tan\theta = \frac{1}{\sqrt{5}}$, $\theta = 24.1(°)$ (0.421 radians)	A1	
		[12]	

Question Number	Scheme	Marks	Notes
4a	Equation of motion: $900a = \frac{22500}{v} - 25v$	M1	Requires all three terms. Condone sign errors
		A1	Correct unsimplified equation
	$a = \frac{\frac{22500}{v} - 25v}{900} = \frac{900 - v^2}{36v}$	A1 [3]	Obtain ****Given answer**** with no errors seen
4b	$\frac{dv}{dt} = \frac{900 - v^2}{36v}$	B1	Differential equation in v and t
	$\int \frac{36v}{900 - v^2} dv = \int 1 dt$,	M1	Separate & integrate
	$t = -18\ln\left(900 - v^2\right)(+C)$	A1	
	: $T = -18\ln 500 + 18\ln 800 = 18\ln\frac{8}{5}$	M1	Use limits correctly Dependent?
		A1	Obtain ****Given answer**** with no errors seen
		[5]	
4c	$\frac{900 - v^2}{36v} = v\frac{dv}{dx}$	B1	Differential equation in v and x
	$\int \frac{v^2}{900 - v^2} dv = \int \frac{1}{36} dx$	M1	Separate variables
	$= \int \frac{900}{900 - v^2} - 1 dv = \left(\int \frac{900}{60}\left(\frac{1}{30 - v} + \frac{1}{30 + v}\right) - 1 dv\right)$	M1	Use partial fractions or equivalent
	$15\ln\left\|\frac{30 + v}{30 - v}\right\| - v = \frac{x}{36}(+C)$	A1	
	$15\ln\left(\frac{50}{10} \times \frac{20}{40}\right) - (20 - 10) = \frac{x}{36}$	M1	Use limits and solve for x Dependent?
	$x = 135$ (m) $\quad (540\ln 2.5 - 360)$	A1	
		[6]	
		(14)	

Question Number	Scheme	Marks	Notes
5a			Extension in AP: $2+x$, Extension in BP: $3+\frac{1}{4}\sin 2t-x-1$
	$T_1=\frac{12(2+x)}{1}$	B1	Force towards A
	$T_2=12\left(2+\frac{1}{4}\sin 2t-x\right)$	B1	Force towards B
	$1.5\frac{d^2x}{dt^2}=T_2-T_1=3\sin 2t-24x$	M1	Form equation of motion of P. Requires derivative and both tensions, but condone sign errors.
		A1	
	$\frac{d^2x}{dt^2}+16x=2\sin 2t$	A1	Obtain *****given answer***** with no errors seen.
		[5]	
5b	$t=0, x=0 \quad \Rightarrow C=0$	B1	
	$t=0, \dot{x}=0=4D\cos 4t+\frac{1}{3}\cos 2t$	M1	
	$D=-\frac{1}{12}$	A1	
	$\dot{x}=0 \Rightarrow \cos 4t=\cos 2t$	M1	At rest: set $\dot{x}=0$
	$2\cos^2 2t-1=\cos 2t$		
	$\cos 2t=1,-\frac{1}{2} \quad 2t=\frac{2\pi}{3}, \quad t=\frac{\pi}{3} \quad (1.05)$	A1	Not $\frac{1}{2}\cos^{-1}\left(\frac{-1}{2}\right)$?
		[5]	

Question Number	Scheme	Marks	Notes
6a			
	GPE of the ring: $-2mgr\cos 2\theta$	B1	
	GPE of suspended particles: $-\sqrt{6}mg\left(L_1-a\right)-\sqrt{6}mg\left(L_2-b\right)$	M1	Expression of the correct structure involving their L_1, L_2, a and b
	$a=2r\sin(45-\theta)=\dfrac{2r}{\sqrt{2}}\left(\cos\theta-\sin\theta\right)$	A1	Correct expression for BR in terms of r and θ Accept $r\sqrt{2\left(1-\sin 2\theta\right)}$
	$b=2r\cos\left(45-\theta\right)=\dfrac{2r}{\sqrt{2}}\left(\cos\theta+\sin\theta\right)$	A1	Correct expression for AR in terms of r and θ Accept $r\sqrt{2\left(1+\sin 2\theta\right)}$
	GPE of system: $-\sqrt{6}mg\left(L_1-a\right)-\sqrt{6}mg\left(L_2-b\right)-2mgr\cos 2\theta$	DM1	Add the three components. Dependent on the previous M
	$=2\times\dfrac{2r}{\sqrt{2}}\cos\theta\times\sqrt{6}mg-2mgr\cos 2\theta$ + constant		
	$=2mgr\left(2\sqrt{3}\cos\theta-\cos 2\theta\right)+$ constant	A1	Simplify to the **given answer**
		[6]	

Question Number	Scheme	Marks	Notes
6b	$\frac{dV}{d\theta} = -4\sqrt{3}\ mgr \text{ in }\theta + 4mgr\sin 2\theta$	M1	Differentiate
	In equilibrium: $\frac{dV}{d\theta} = 0 = 4mgr\sin\theta\left(-\sqrt{3}+2\cos\theta\right)$	M1	Set $\frac{dV}{d\theta}=0$ and solve for θ
	$\theta = \pm\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pm\frac{\pi}{6}(=\pm 0.52)$	A1	
	or $\theta = 0$	B1	
		[4]	
6c	$\frac{d^2V}{d\theta^2} = -4\sqrt{3}mgr\cos\theta + 8mgr\left(\cos^2\theta - \sin^2\theta\right)$	M1	Second derivative - needs to be the full expression.
	$\frac{d^2V}{d\theta^2} = mgr\left(-4\sqrt{3}\times\frac{\sqrt{3}}{2} + 8\left(\frac{3}{4}-\frac{1}{2}\right)\right) = -2mgr < 0$	M1	Substitute $\theta = \frac{\pi}{6}$
	So equilibrium is unstable	A1	No errors seen
		[3]	
		(13)	

Question Number	Scheme	Marks	Notes
7a	Resolve parallel to barrier - condone sin/cos confusion	M1	
	$u\cos 60 = v\cos\theta$	A1	
	Resolve perpendicular to the barrier - condone sin/cos confusion	M1	
	$eu\sin 60 = v\sin\theta$	A1	
	$v^2 = u^2\cos^2 60 + e^2u^2\sin^2 60 = \frac{u^2}{4} + \frac{3u^2}{16} = \frac{7u^2}{16}$	M1	Eliminate θ and solve for v.
	$v = \frac{\sqrt{7}}{4}u$	A1	Obtain given answer correctly with no errors seen
		[6]	
7b	Angle of approach with BC = 19.1°	B1	
	$v\cos 19.1 = w\cos\phi$	M1	Components parallel to BC
	$\frac{1}{2}v\sin 19.1 = w\sin\phi$	M1	Components perpendicular to BC
		A1	Equations correct for their 19.1
	Form equation in v and ϕ	M1	Square and add or divide to find $\tan\phi$
	$w^2 = v^2\left(\frac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$	A1	$(\phi = 9.83°)$ Follow their 19,1?
	$0.634u$	A1	
7balt	$\tan\theta = \frac{1}{2}\tan 60$	B1	

Question Number	Scheme	Marks	Notes
	$\tan\alpha = \frac{1}{2}\tan(60-\theta) \quad \left(= \frac{1}{2}\left(\frac{\sqrt{3}-\frac{1}{2}\sqrt{3}}{1+\sqrt{3}.\frac{1}{2}\sqrt{3}}\right) = \frac{\sqrt{3}}{10}\right)$	M1	
		A1	
	$v\cos(60-\theta) = w\cos\alpha$	M1	
	$v\left(\frac{1}{2}.\frac{2}{\sqrt{7}} + \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{\sqrt{7}}\right) = w\frac{10}{\sqrt{103}}\left(= v\frac{5}{2\sqrt{7}}\right)$	M1	
		A1	
	$w = \frac{\sqrt{103}}{4\sqrt{7}}v = \frac{\sqrt{103}}{4\sqrt{7}}.\frac{\sqrt{7}}{4}u = \frac{\sqrt{103}}{16}u \qquad (0.634u)$	A1	
		[7]	

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